



Practical XRD with Confidence



Episode 4 – Beyond Diffraction: Exploring
Nanostructure with SAXS, WAXS, and X-Ray Reflectivity

Wednesday, June 17, 2026, at 1 pm CDT

Presenter: Keisuke Saito, PhD

Host: Tom Concolino, PhD





Practical XRD with Confidence

Episode 4 – Beyond Diffraction: Exploring
Nanostructure with SAXS, WAXS, and X-Ray Reflectivity

- *You will be muted during the workshop*
- *You can ask questions using the Q&A tool.*
- *You should hear music if your sound is working*



Presenter:
Keisuke Saito
Director, Application Science



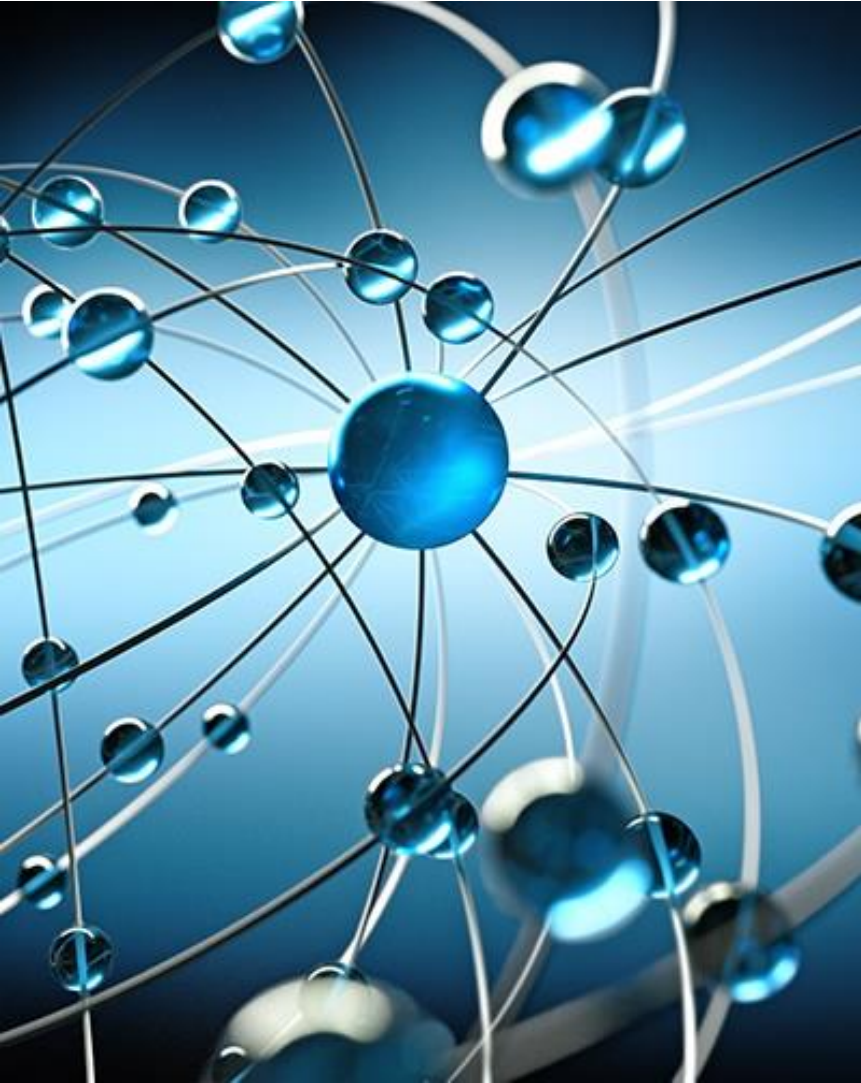
Host:
Tom Concolino
Sales Manager, XRD

You can ask questions during the presentation. Please use the Q&A to ask questions.



Recording will be available tomorrow.



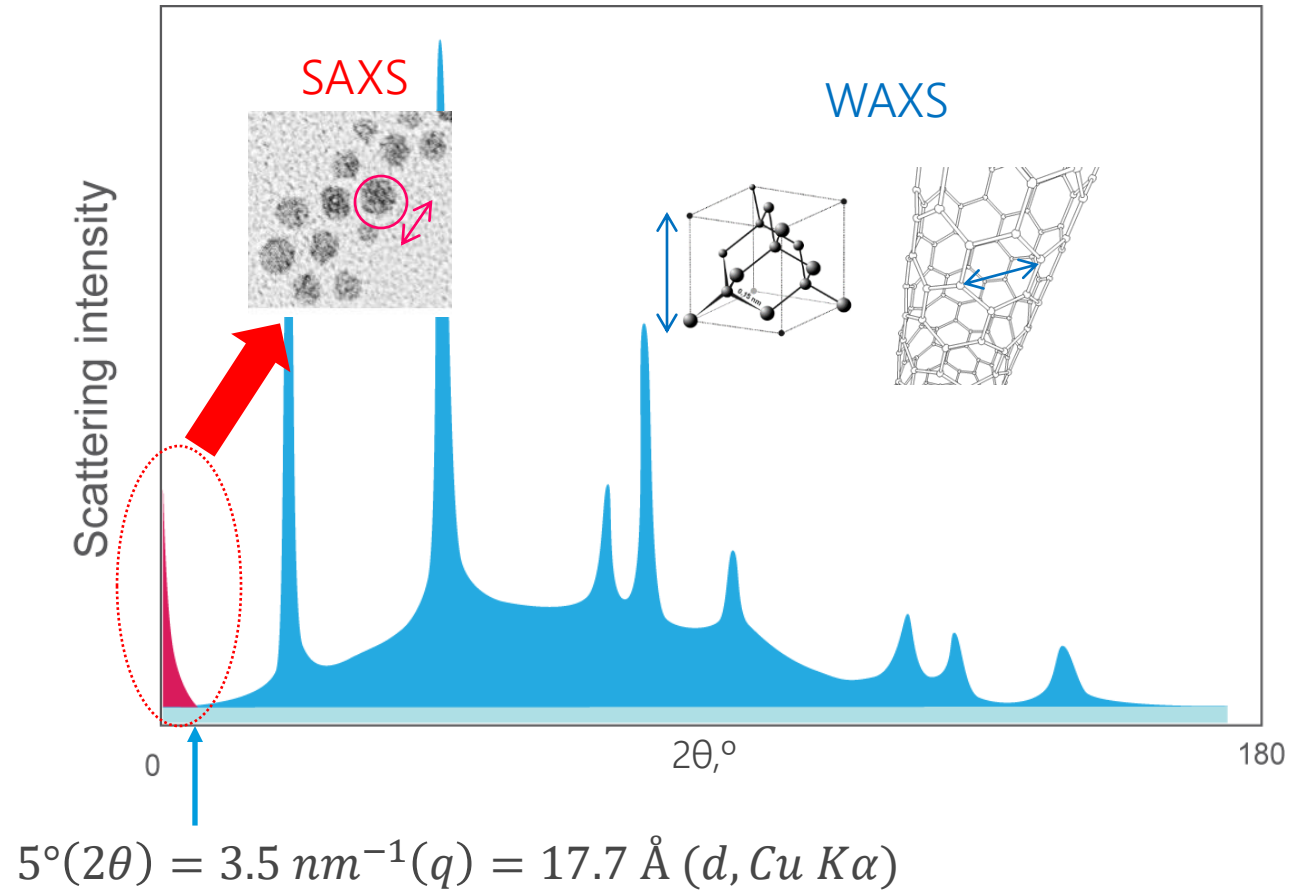


What I will cover today:

1. X-ray scattering fundamentals
2. SAXS fundamentals
3. SAXS setup & sample preparation
4. Sizing nanostructures with SAXS
5. Case study: oriented polymers
6. Case study: mesoporous silica
7. XRR fundamentals
8. XRR modeling & fitting

1. X-ray scattering fundamentals

SAXS & WAXS



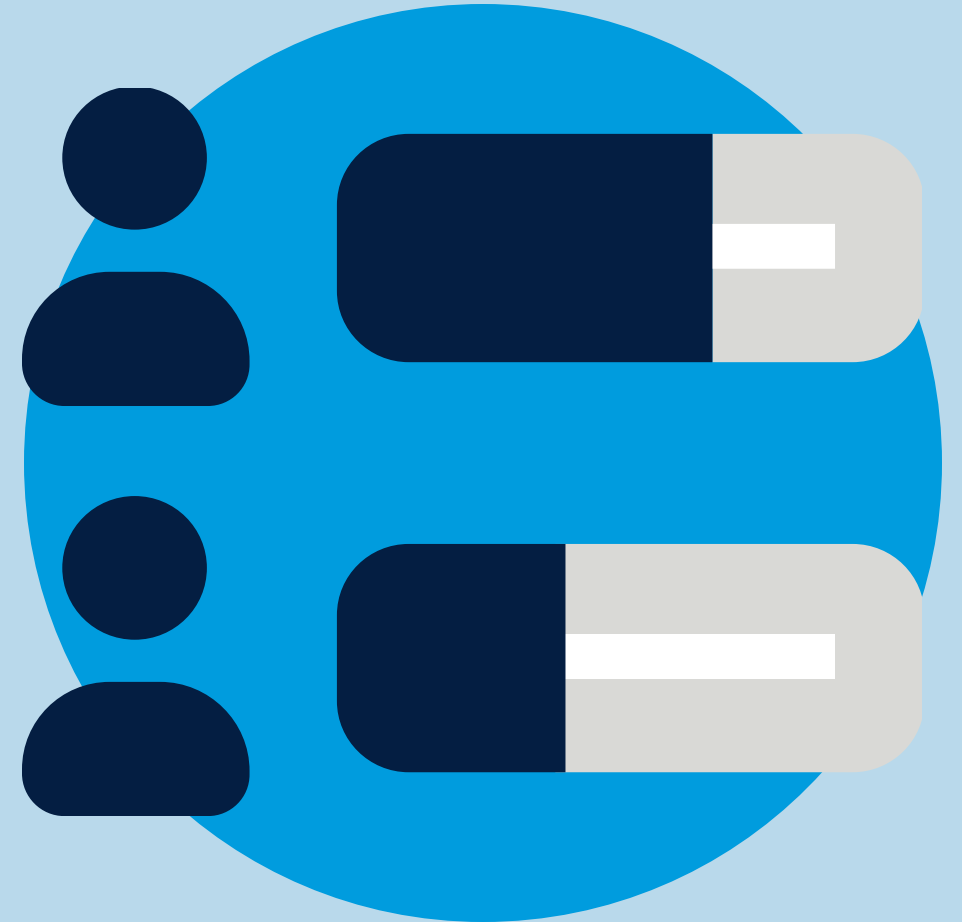
Comparison of X-ray scattering and diffraction techniques

Technique	Structure size	What causes signal?	Information obtained
SAXS	1 ~ 1,000 nm	Electron density fluctuations	Particle size, pores, aggregates
WAXS	0.1 ~ 10 nm	Atomic correlations	Crystal structure, phase, crystallinity, local order
XRD	0.1 ~ 1 nm	Bragg diffraction	Crystal structure, phase
XRR	0.1 ~ 1,000 nm	Reflection from interfaces	Thickness, density, roughness

Polling Question

#1 Who has previously performed SAXS, WAXS, XRD or XRR?

1. SAXS
2. WAXS
3. XRD
4. XRR



2. SAXS fundamentals

Scattering vector

XRD

- $|\mathbf{k}| = \frac{1}{\lambda}$
- $K = 2 \cdot |\mathbf{k}| \cdot \sin \theta = \frac{2}{\lambda} \cdot \sin \theta = \frac{1}{d_{hkl}} = \mathbf{g}_{hkl}$

XRD: reciprocal spacing $1/d$.

SAXS / WAXS

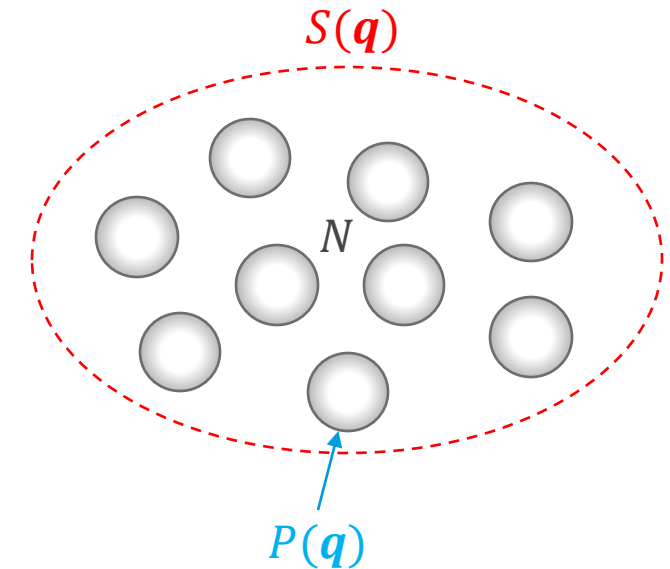
- $|\mathbf{k}| = \frac{2\pi}{\lambda}$
- $\mathbf{q} = 2 \cdot |\mathbf{k}| \cdot \sin \theta = \frac{4\pi}{\lambda} \cdot \sin \theta = \frac{2\pi}{d_{hkl}}$

SAXS/WAXS: Fourier-transform convention \mathbf{q}

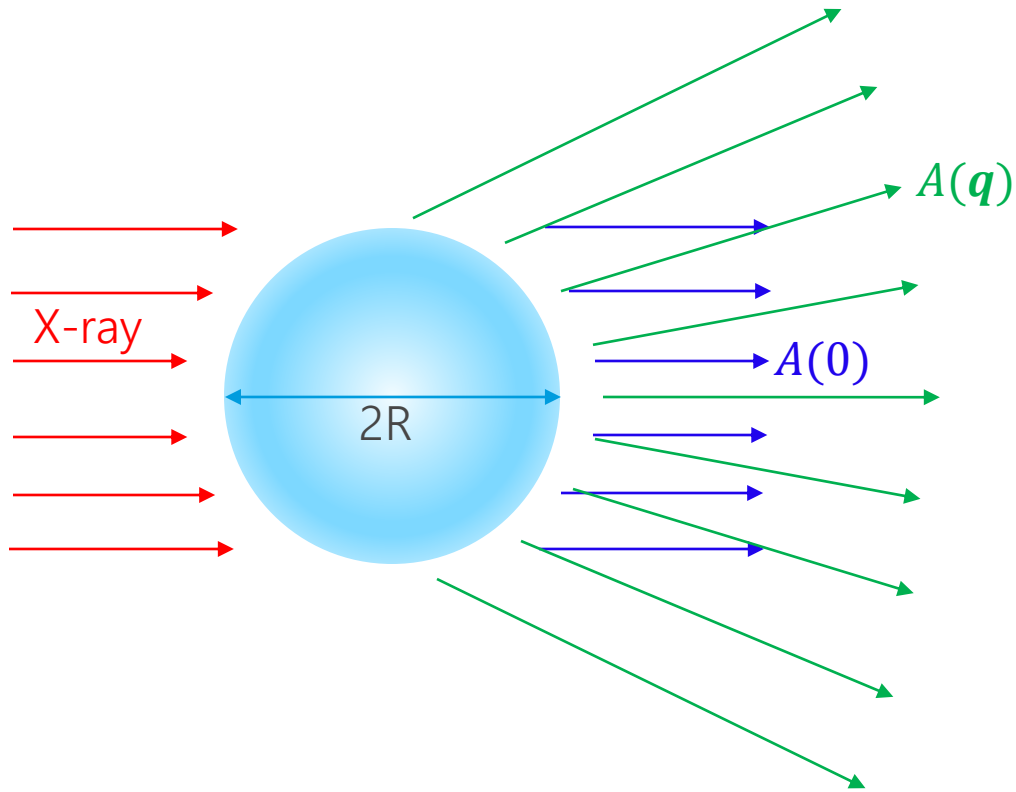
λ : wavelength of X-ray
 \mathbf{K}, \mathbf{q} : scattering vector
 \mathbf{k} : X-ray wave vector
 \mathbf{g}_{hkl} : reciprocal lattice vector
 θ : Bragg angle
 d_{hkl} : interplanar spacing

SAXS: scattering intensity - isotropic case

- $I(\mathbf{q}) \propto N \cdot P(\mathbf{q}) \cdot S(\mathbf{q})$
 - N : number of nano particles
 - $P(\mathbf{q})$: form factor
 - $S(\mathbf{q})$: structure factor
- What shape/size are the particles? → $P(\mathbf{q})$
- How are they arranged? → $S(\mathbf{q})$



Form factor $P(q)$ for sphere



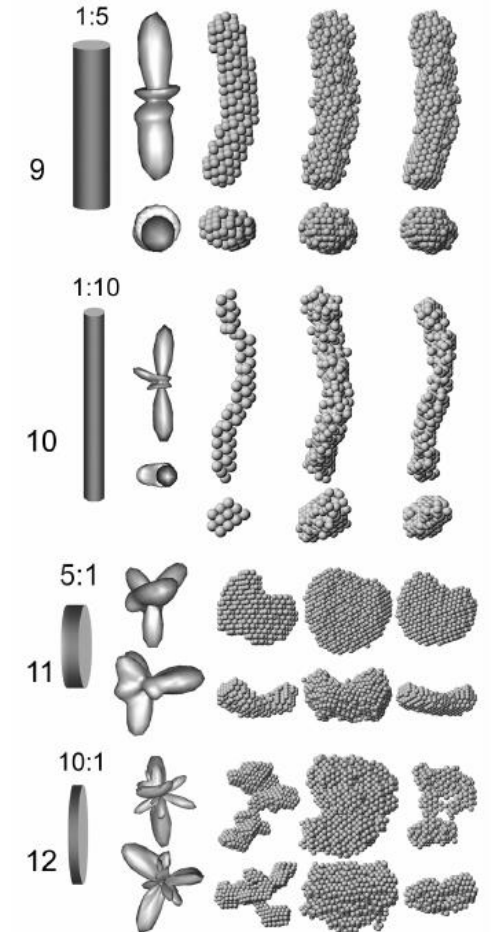
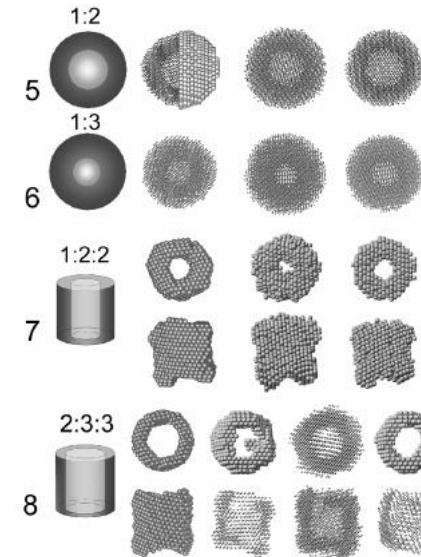
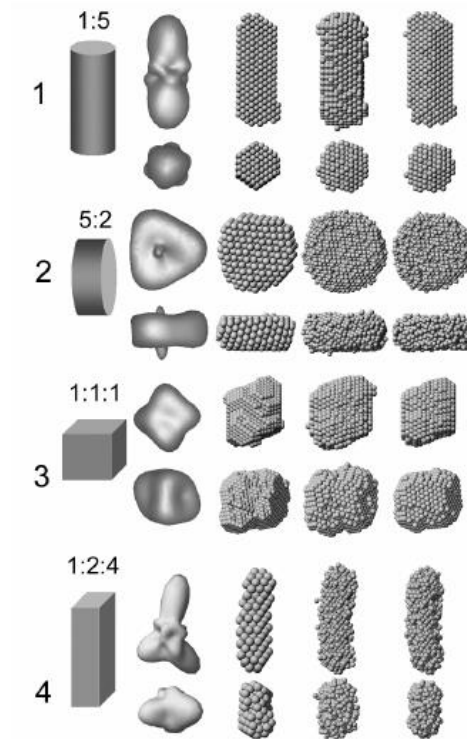
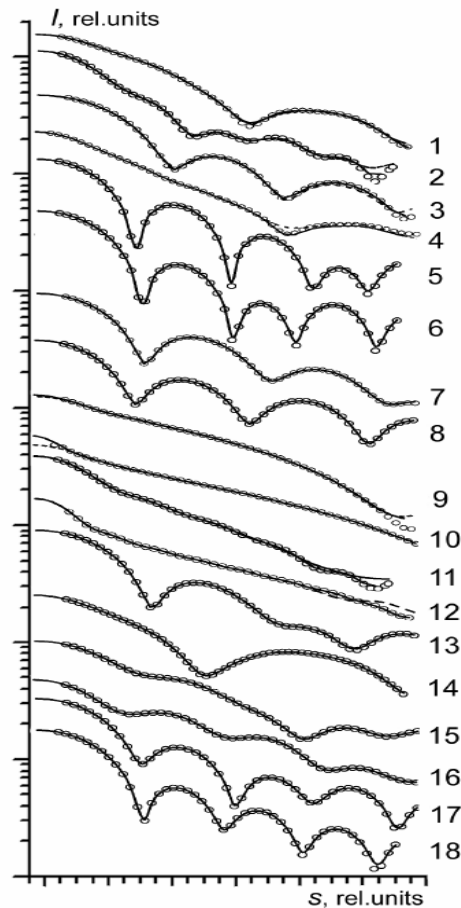
- Scattering amplitude for sphere

- $A(\mathbf{q}) = \int \rho(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} d\mathbf{r}$
 $= \rho V \left[\frac{\sin(qR) - qR \cos(qR)}{(qR)^3} \right]$

- Form factor

- $P(q) = \left| \frac{A(q)}{A(0)} \right|^2 = \left[3 \frac{\sin(qR) - qR \cos(qR)}{(qR)^3} \right]^2$

Form factor $P(q)$ for other shapes

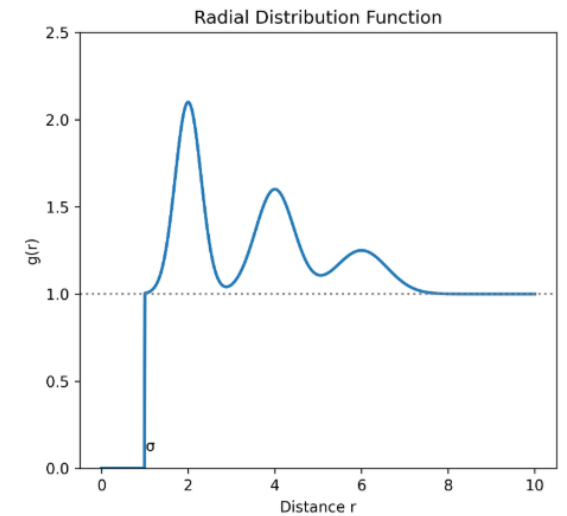
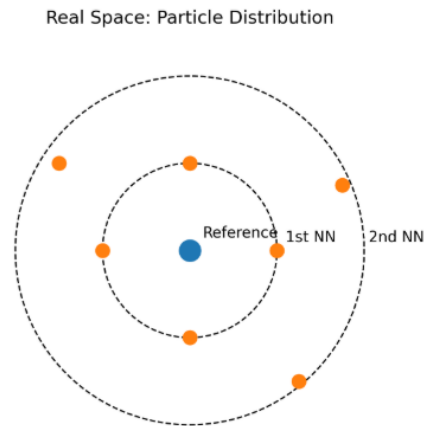


[V. Volkov, D. Svergun, J.Appl.Cryst. \(2003\) 36, 860-864.](#)

Structure factor $S(q)$

- $S(q) = 1 + \int_v (n(\mathbf{r}) - n_0) e^{iq \cdot r} d\mathbf{r}$
 - $n(\mathbf{r})$: particle count density function
 - n_0 : average particle density
- $n(r)$
 - Radial distribution function (RDF)
 - Percus-Yevick (PY) model for isotropic case

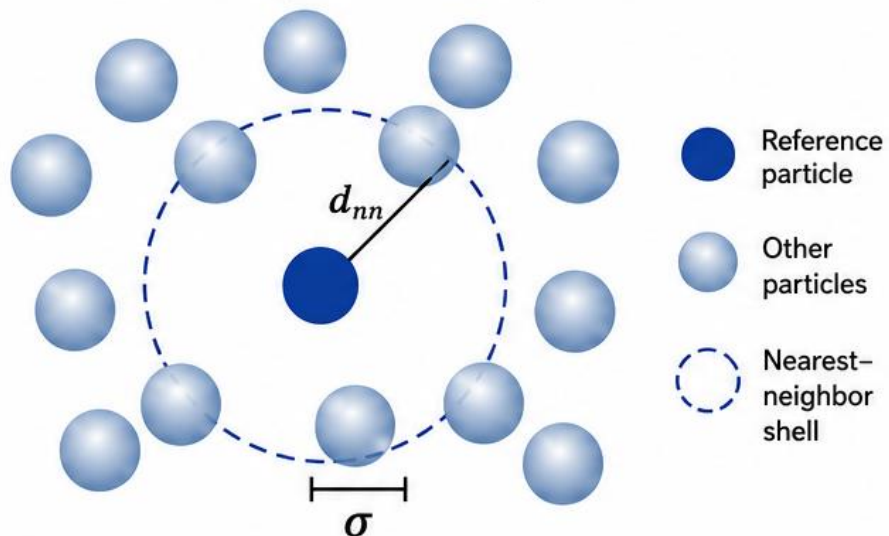
RDF



PY model

1. Real-space picture

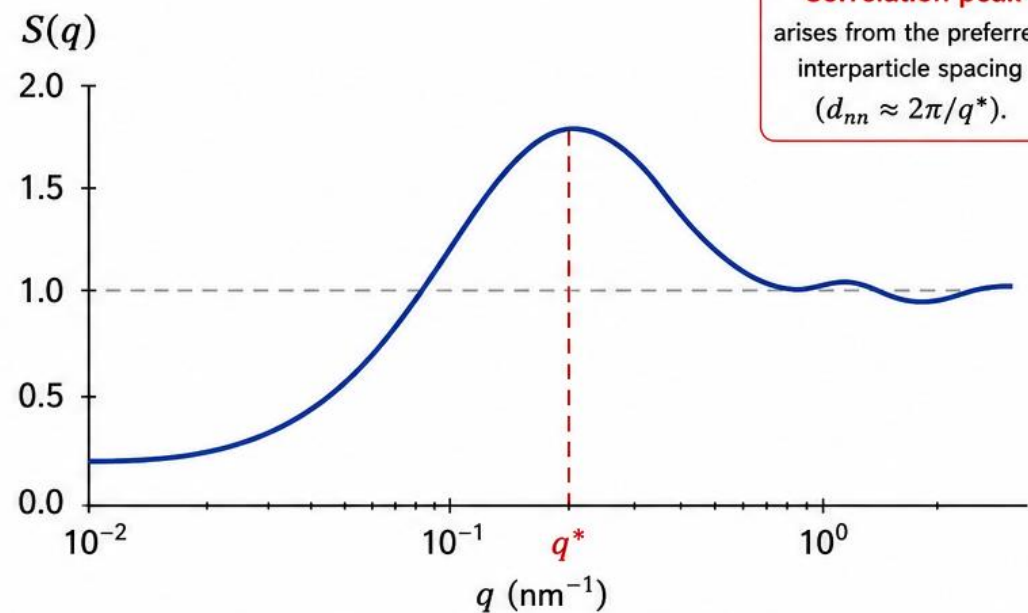
Hard spheres cannot overlap (excluded volume).
 Particles are arranged with short-range order.



- σ = hard-sphere diameter (particle diameter)
- d_{nn} = average center-to-center distance to nearest neighbor
- η = packing fraction (volume fraction occupied by spheres)

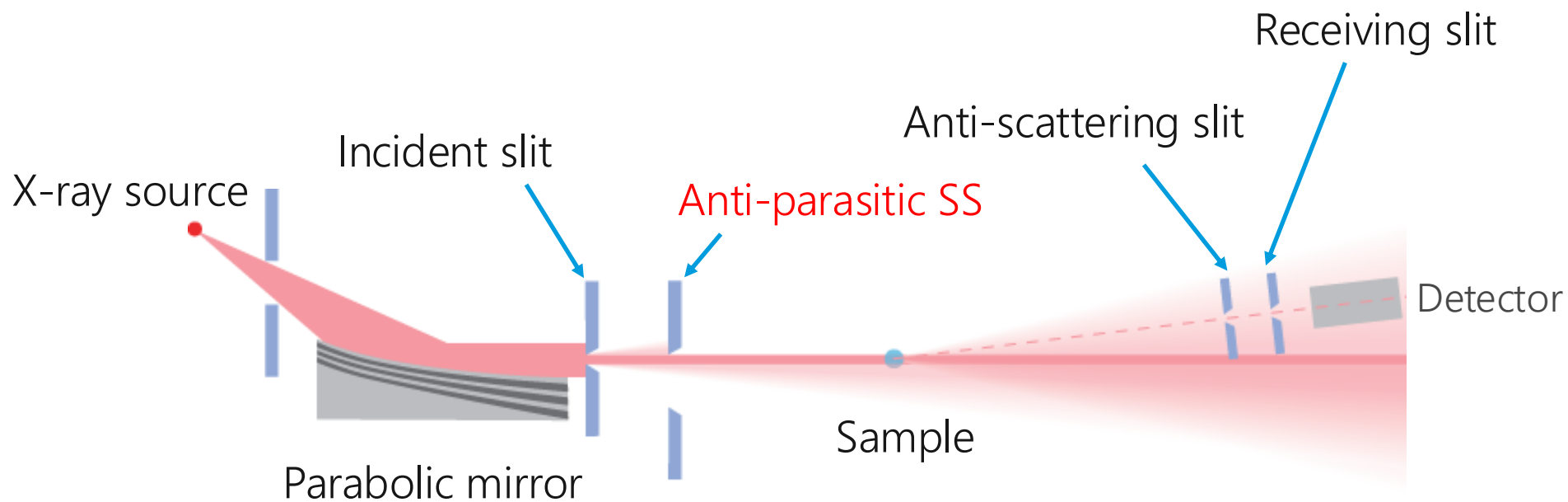
2. Structure factor $S(q)$ predicted by PY model

A broad peak appears at q^* due to the preferred interparticle spacing.

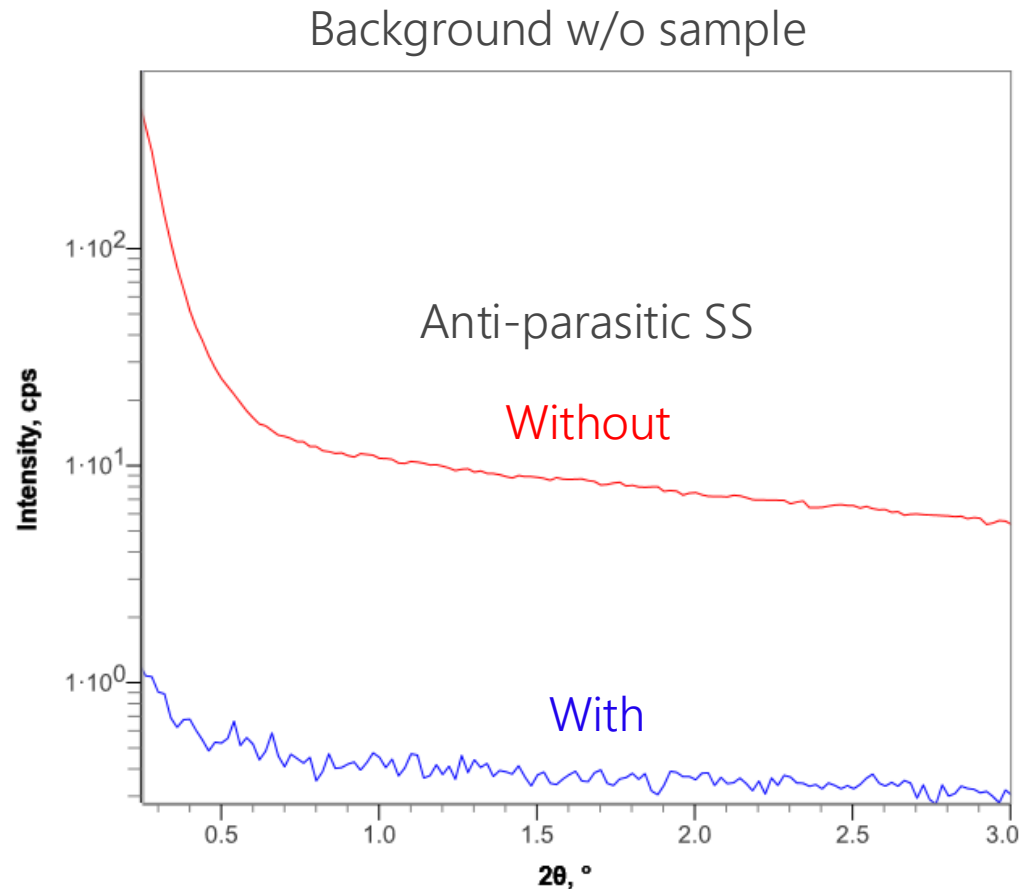


3. SAXS setup & sample preparation

Instrument configurations

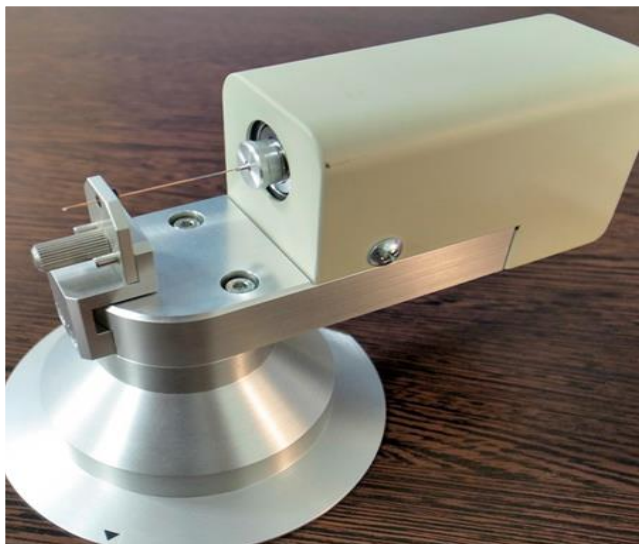


Parasitic scattering



- Parasitic scattering is unwanted low-angle scattering caused by optics such as slits.

Sample preparation



- Fused silica capillary
- Liquid, powder sample
- 0.3, 0.5, 0.7, 0.9, 1.0 mmφ

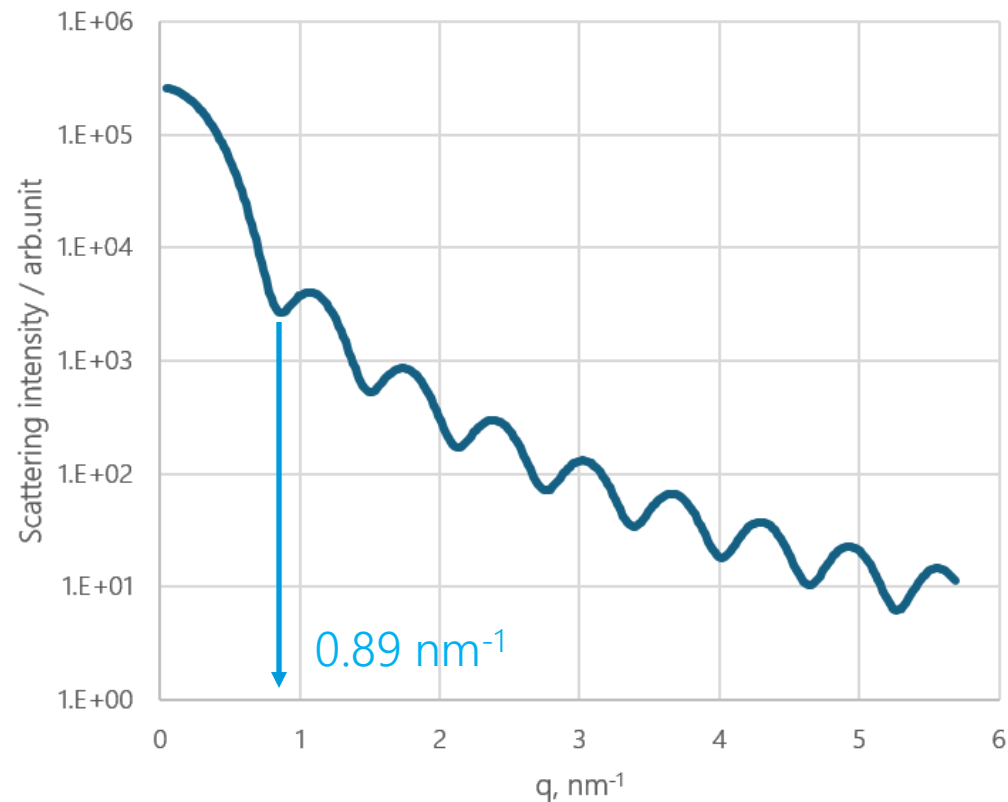


- Foil-transmission
- Powder, liquid, gel sample
- Sample to be placed between two mylar / Kapton foils

- $20 \% \leq T = \frac{I_{sample}}{I_{no\ sample}} \leq 80 \%$
 - T : transmittance
 - I_{sample} : intensity transmits through sample and holder
 - $I_{no\ sample}$: intensity transmits through holder without sample

4. Sizing nanostructures with SAXS

Particle size estimation (1): peak search



- Sphere model

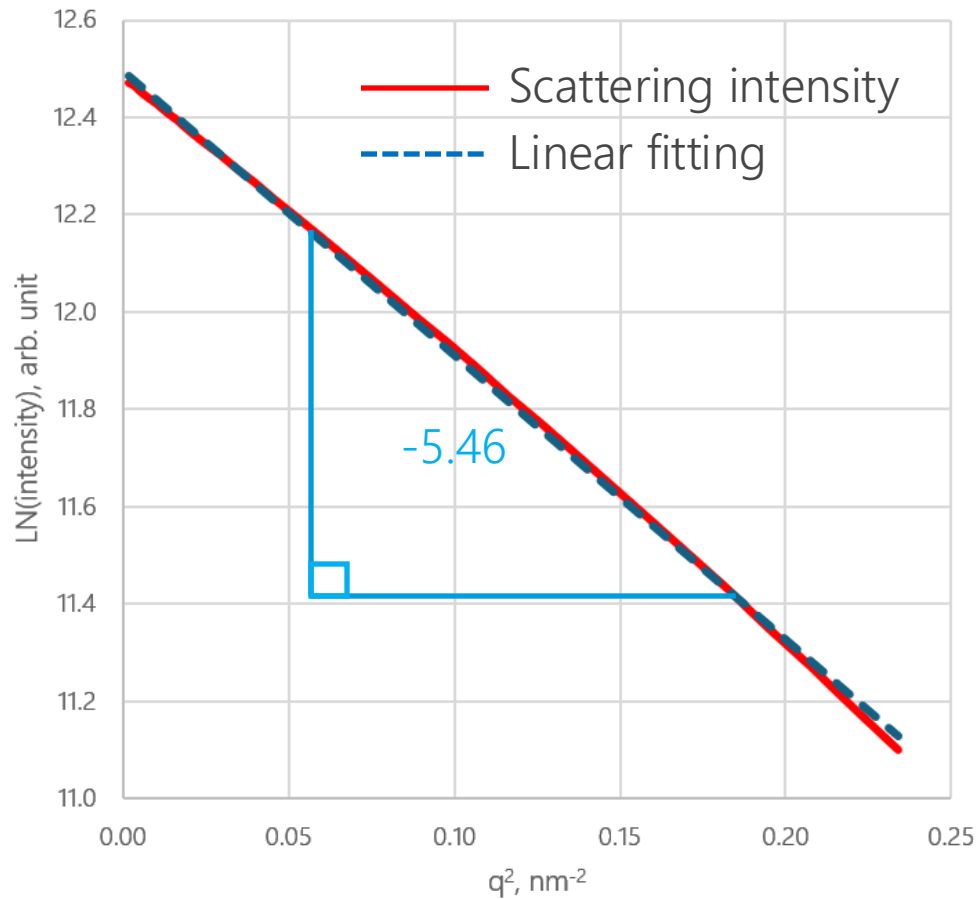
- $P(q) = \left[3 \frac{\sin(qR) - qR \cos(qR)}{(qR)^3} \right]^2$

- 1st minimum $qR \approx 4.493$

- Particle diameter: D (nm)

- $D = 2R = \frac{4.493}{0.89} \sim 10$

Particle size estimation (2): Guinier plot



- In q range, $qR_g < 1$

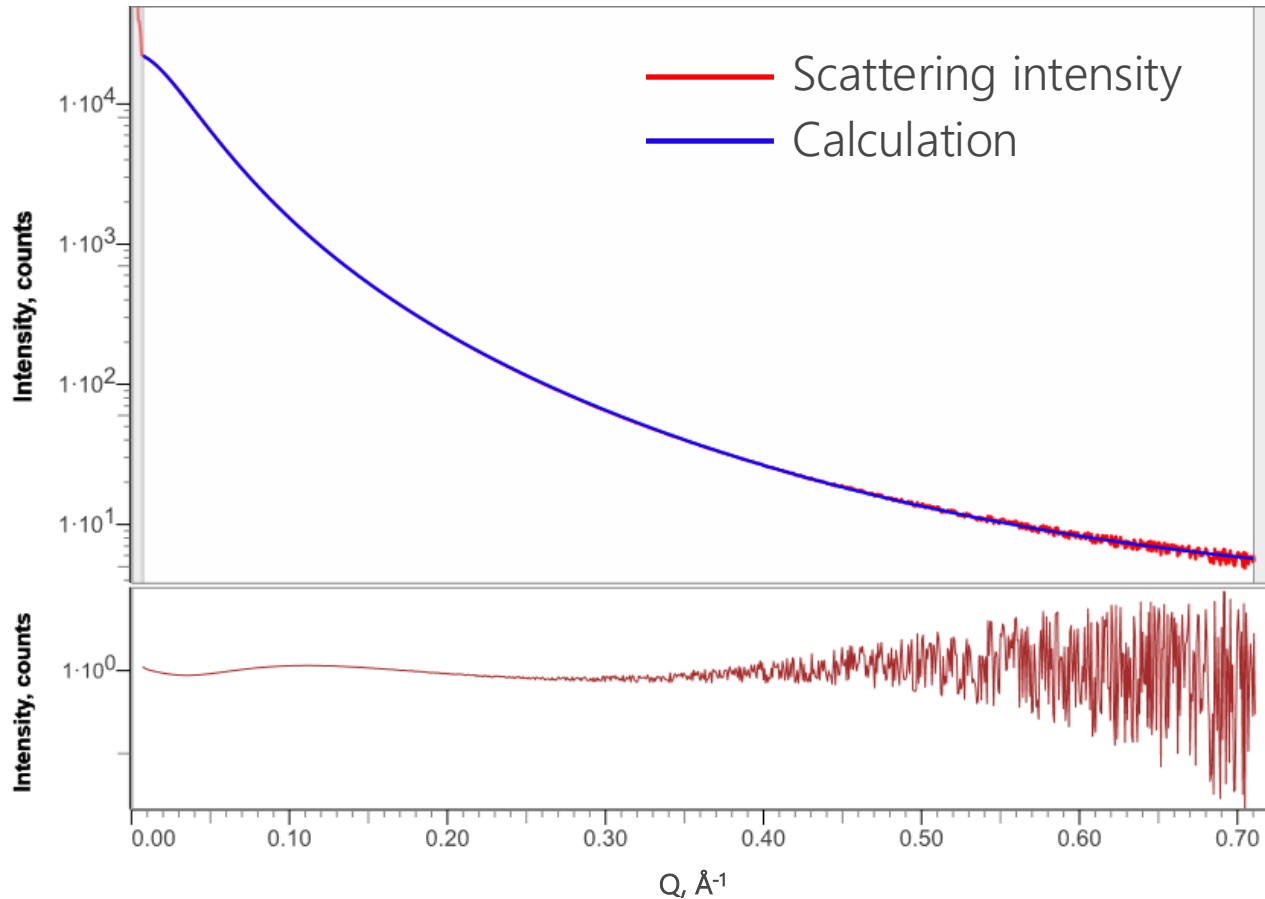
- $I(q) = I(0) \exp\left(-\frac{q^2 R_g^2}{3}\right)$

- R_g : radius of gyration

- $D = 2R = 2\sqrt{\frac{5}{3}} R_g = 2\sqrt{\frac{5}{3}} \sqrt{-3 \cdot \text{slope}}$
 $= 10.4 \text{ nm}$ (sphere)

- $q_{max} < \frac{1}{R_g} = 0.247 \text{ nm}^{-1}$

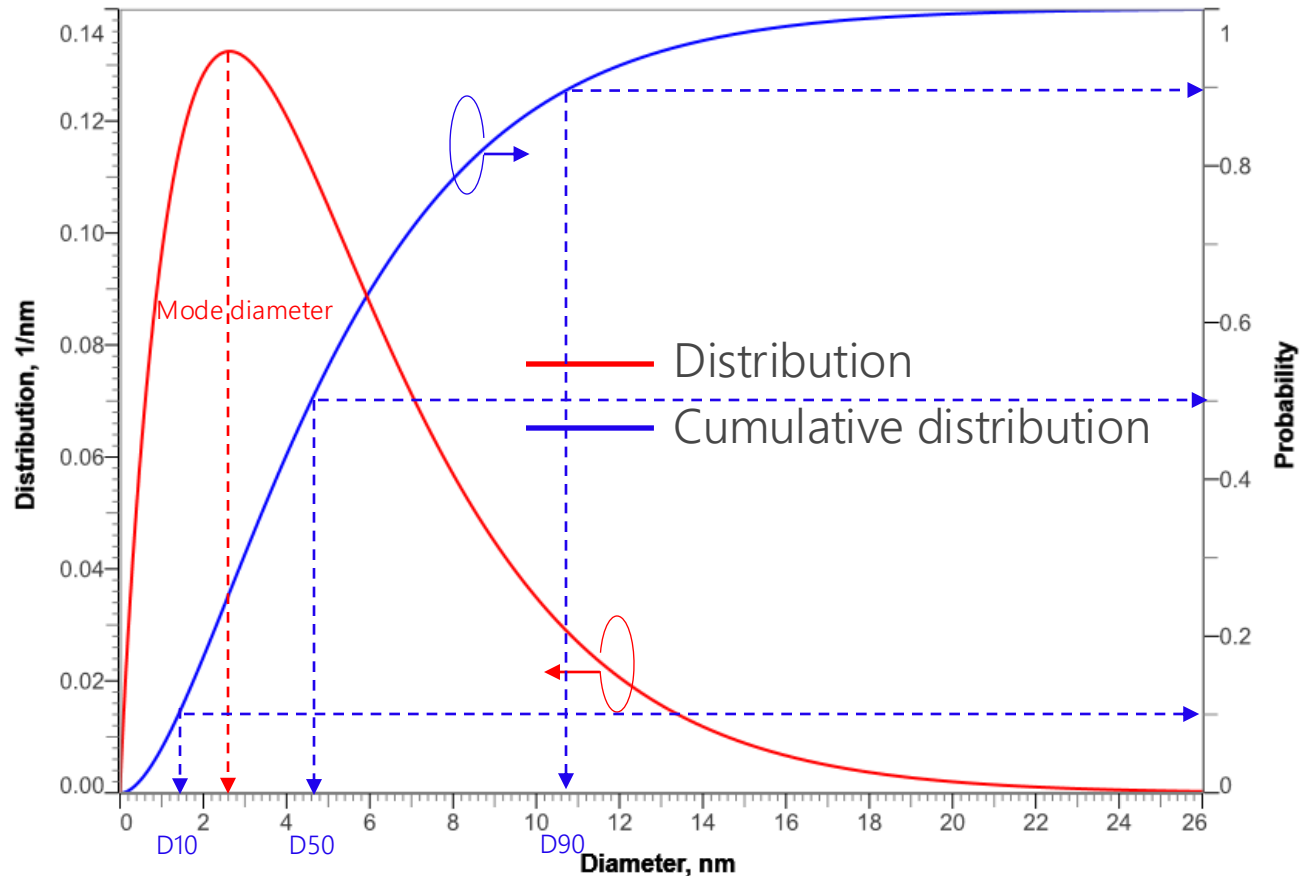
Particle size estimation (3): Model fitting



- Sample
 - Particles: gold (sphere)
 - Matrix: toluene
- Average diameter: 5.5 nm
- Size distribution: 70 %

[Pedersen, J.S., "Analysis of Small-Angle Scattering Data from Colloids and Polymer Solutions: Modeling and Least-Squares Fitting," Advances in Colloid and Interface Science, 70, 171–210 \(1997\).](#)

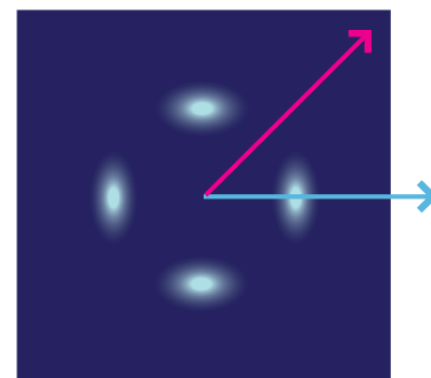
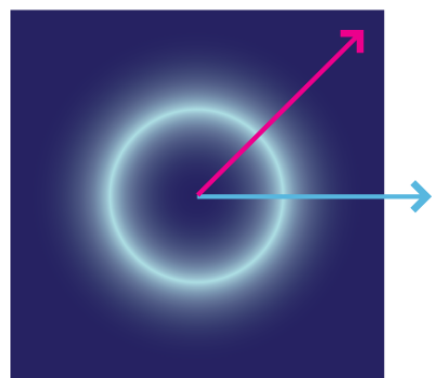
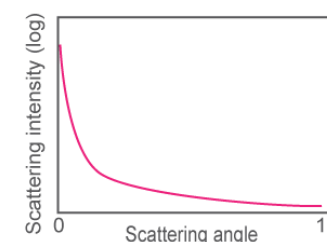
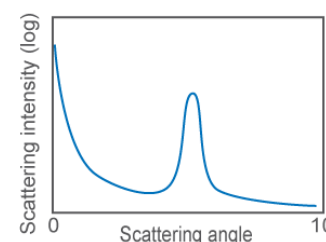
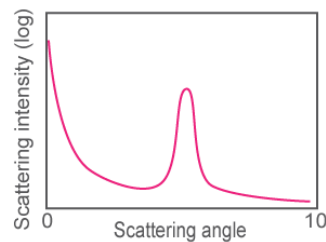
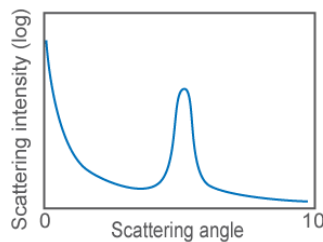
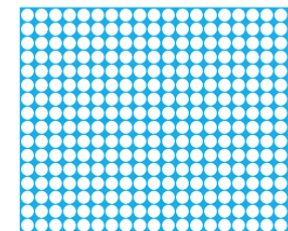
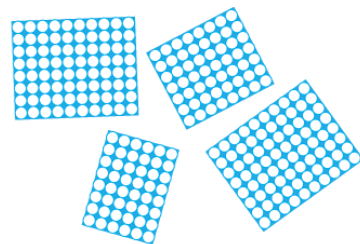
Particle size estimation (3): Model fitting



- Average diameter: 5.5 nm
- Size distribution: 70 %
- Mode diameter: 2.6 nm
- D10: 1.4 nm
- D50: 4.6 nm
- D90: 10.8 nm

5. Case study: oriented polymers

Texture effect



Instrument configurations – 2D SAXS

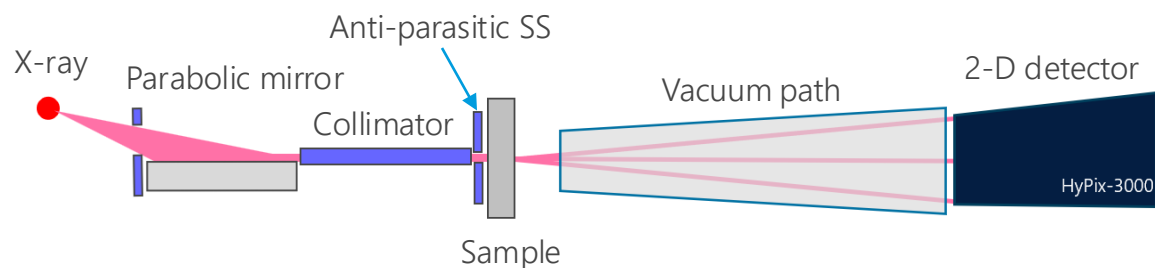
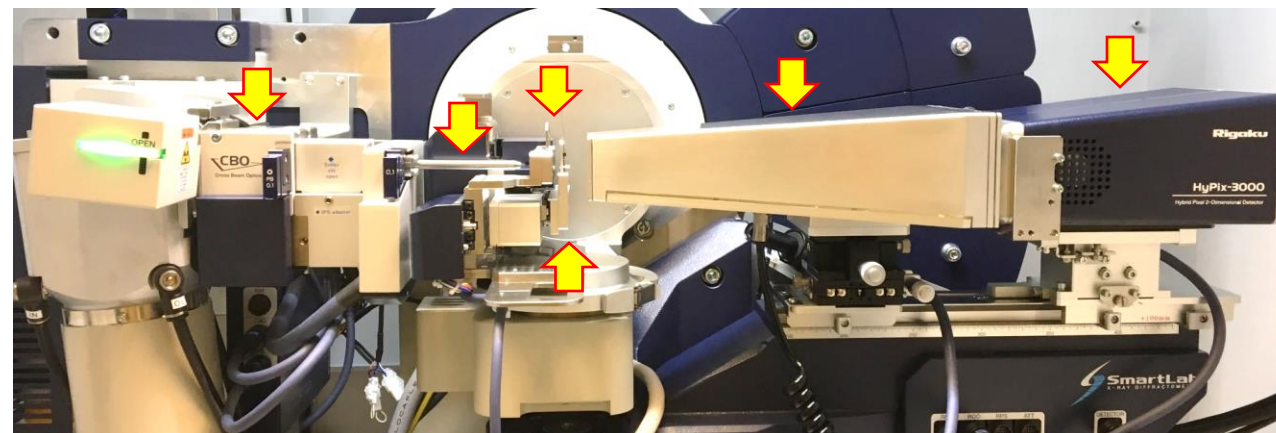
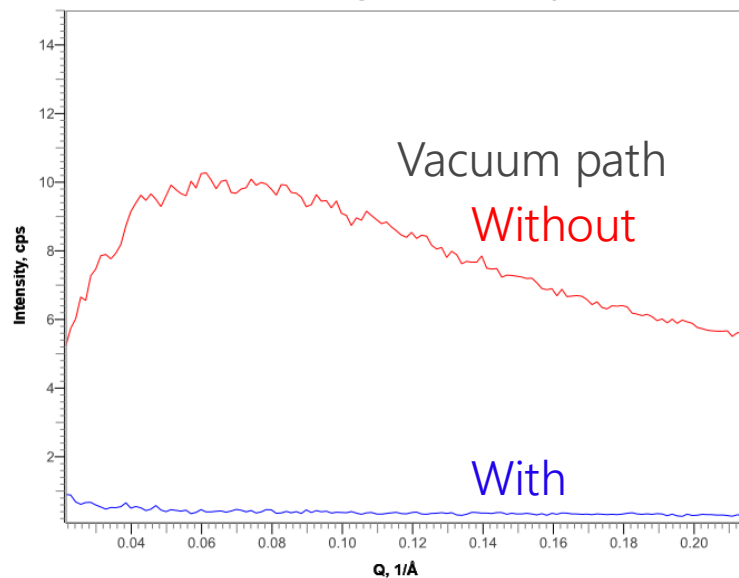
2D-SAXS

$$0.2 < 2\theta < 7.0^\circ$$

$$12.6 < d < 441 \text{ \AA}$$

$$0.014 < Q < 0.498 \text{ \AA}^{-1}$$

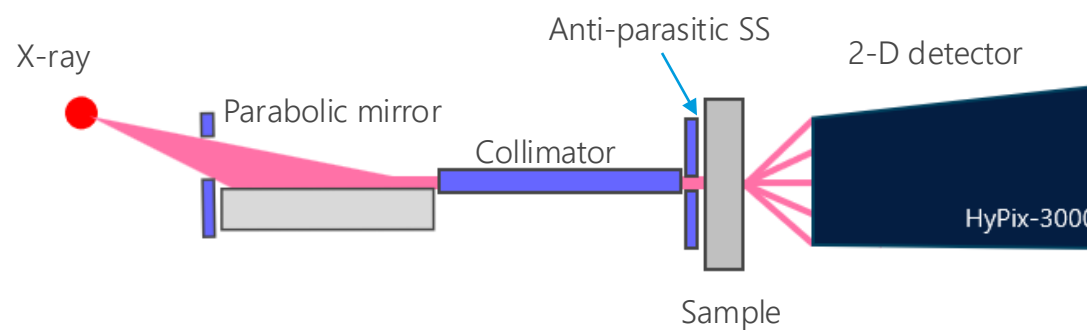
Scattering no-sample



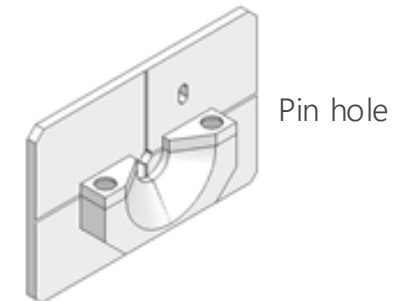
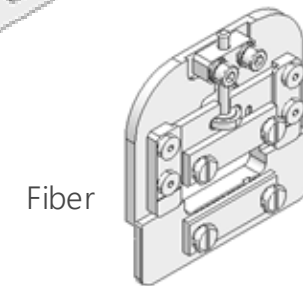
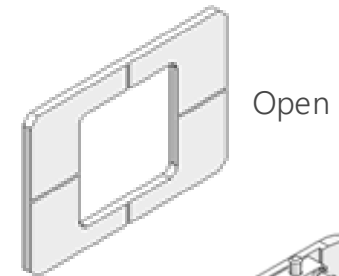
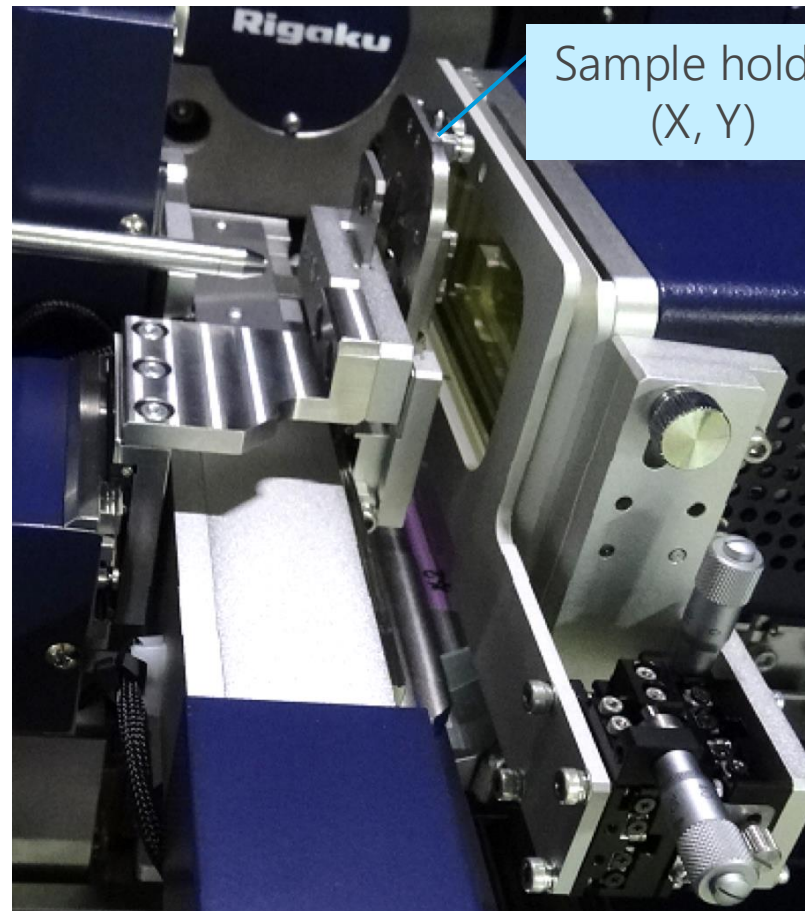
Instrument configurations – 2D WAXS

2D-WAXS

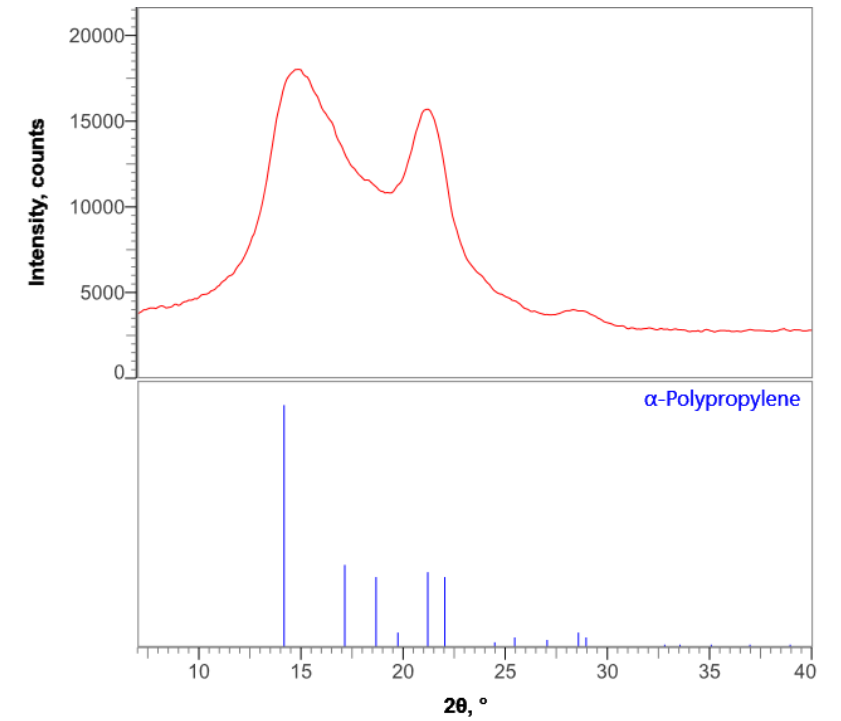
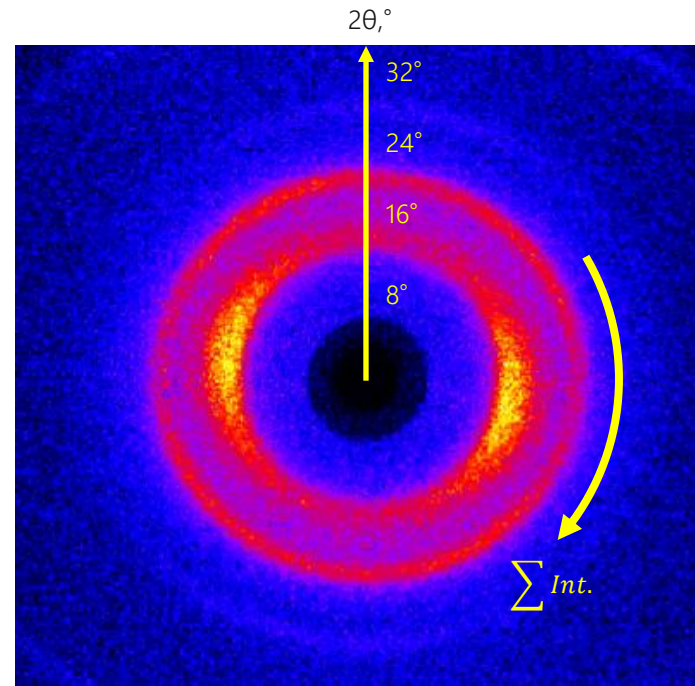
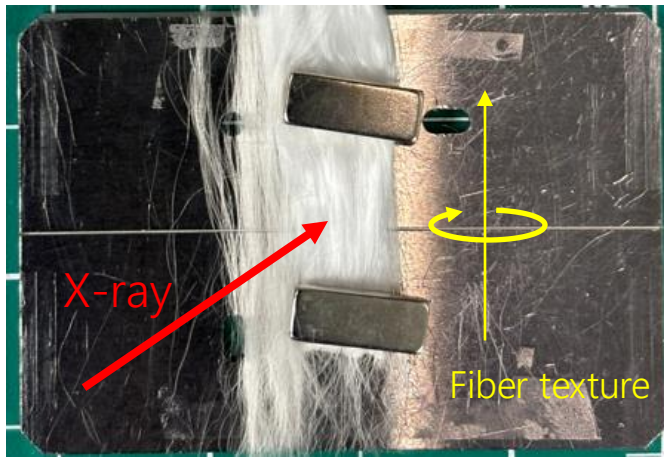
$$6.5 < 2\theta < 35^\circ$$
$$2.56 < d < 13.59 \text{ \AA}$$
$$0.46 < Q < 2.45 \text{ \AA}^{-1}$$



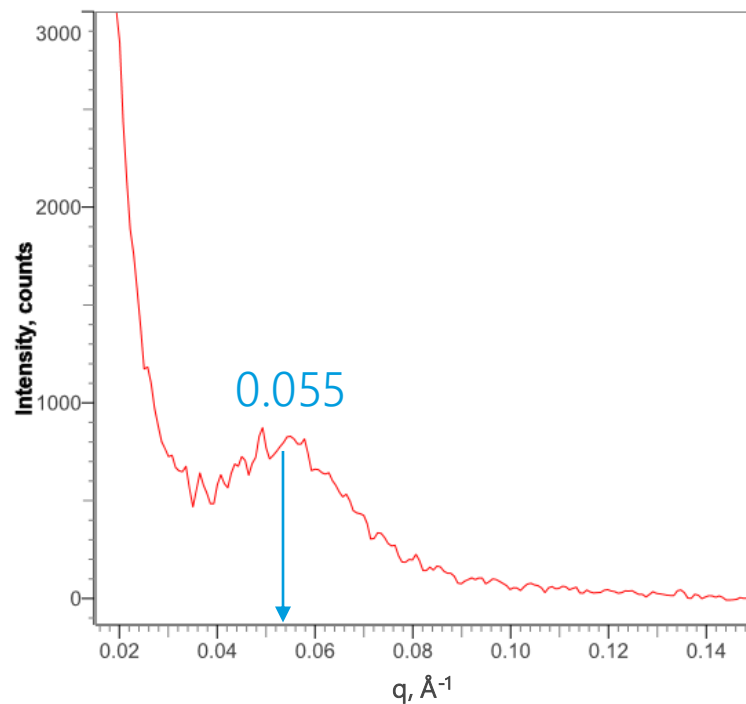
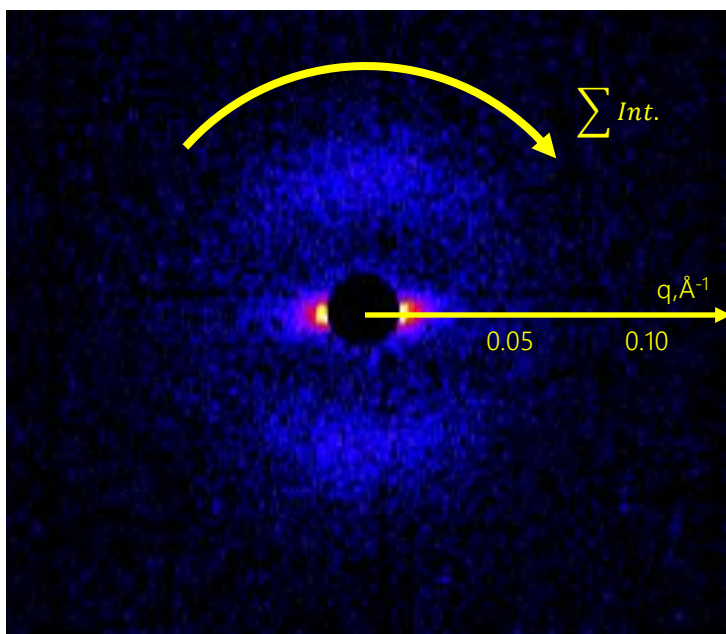
Sample holders



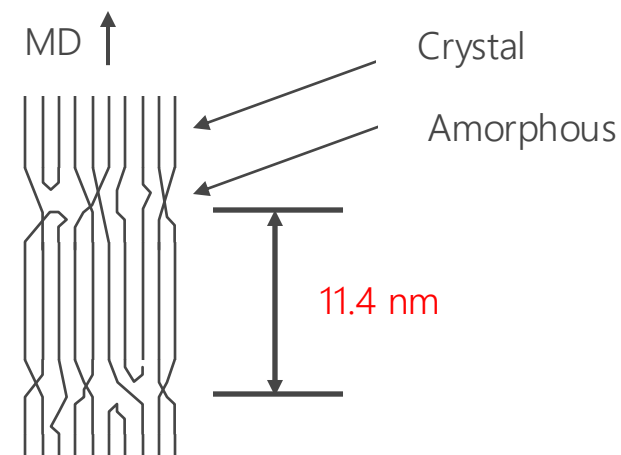
Polymer: WAXS



Polymer: SAXS

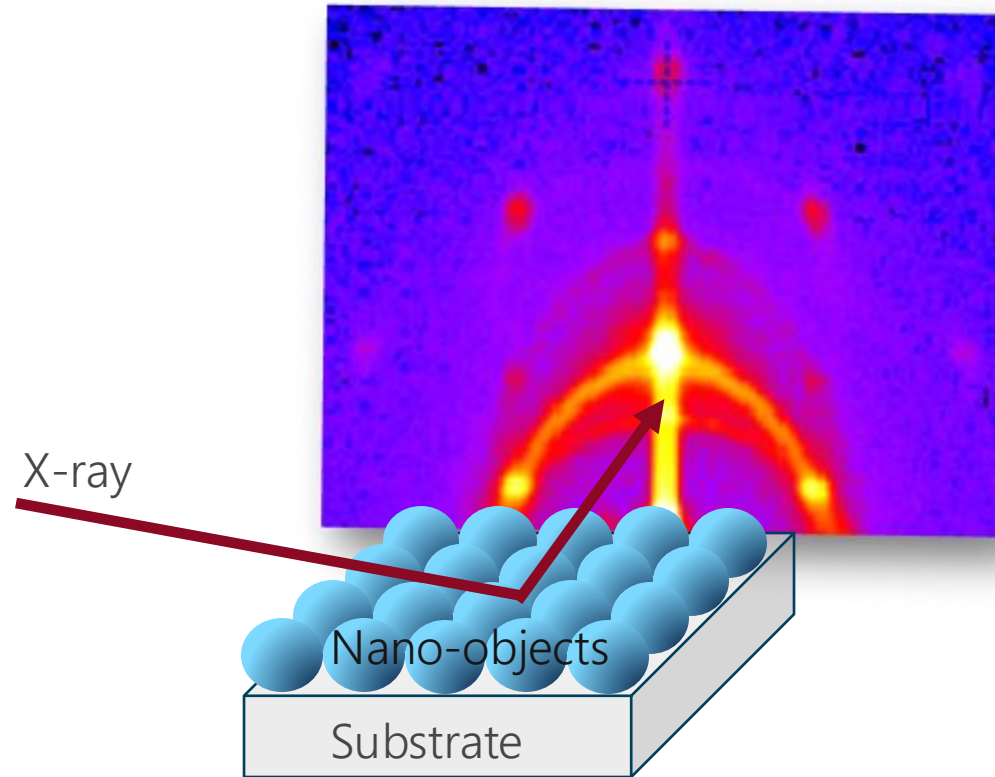


- $$d = \frac{2\pi}{q} = 11.4 \text{ nm}$$

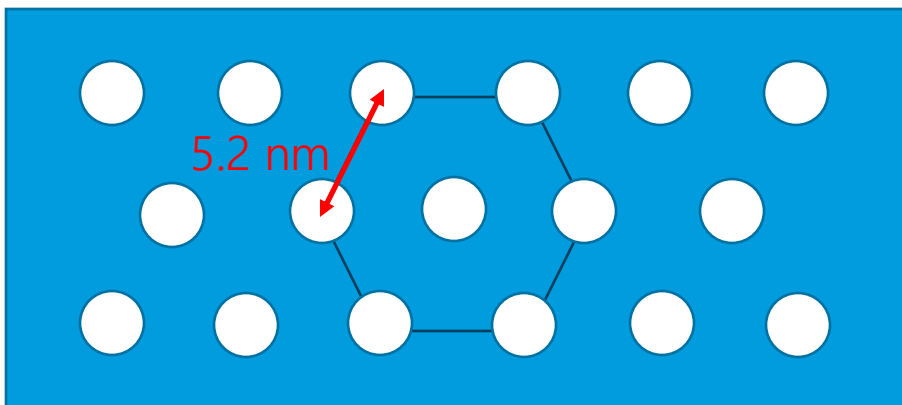
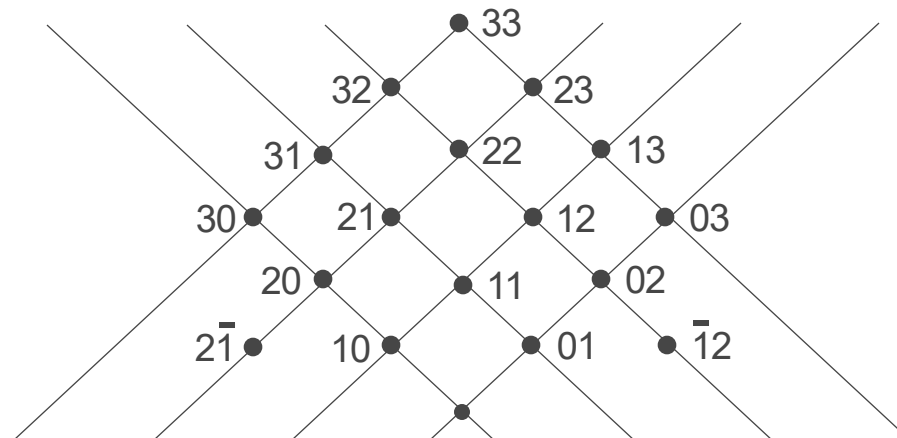
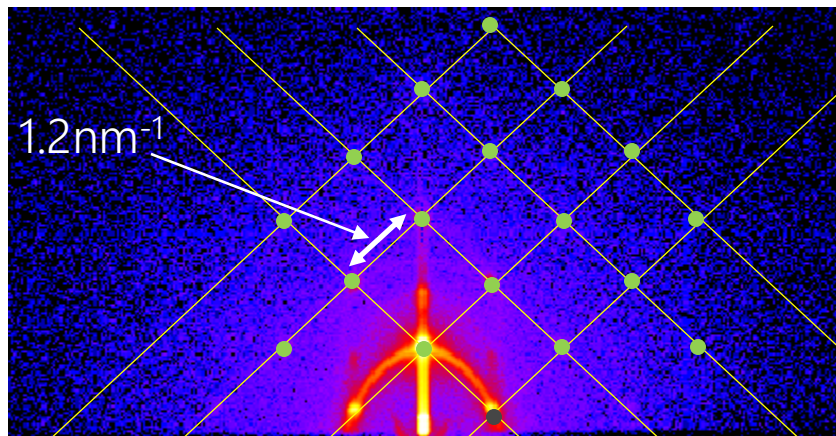


6. *Case study: mesoporous silica*

Mesoporous silica: GIWAXS



Mesoporous silica: GIWAXS



- $$d = \frac{2\pi}{q} = 5.2 \text{ nm}$$

Summary

- SAXS and WAXS provide complementary structural information across multiple length scales.
- SAXS profiles contain quantitative information on particle size, shape, size distribution, and interparticle spacing.
- 2D SAXS/WAXS reveals anisotropy and orientation, enabling characterization of textured materials such as fibers, films, and polymers.

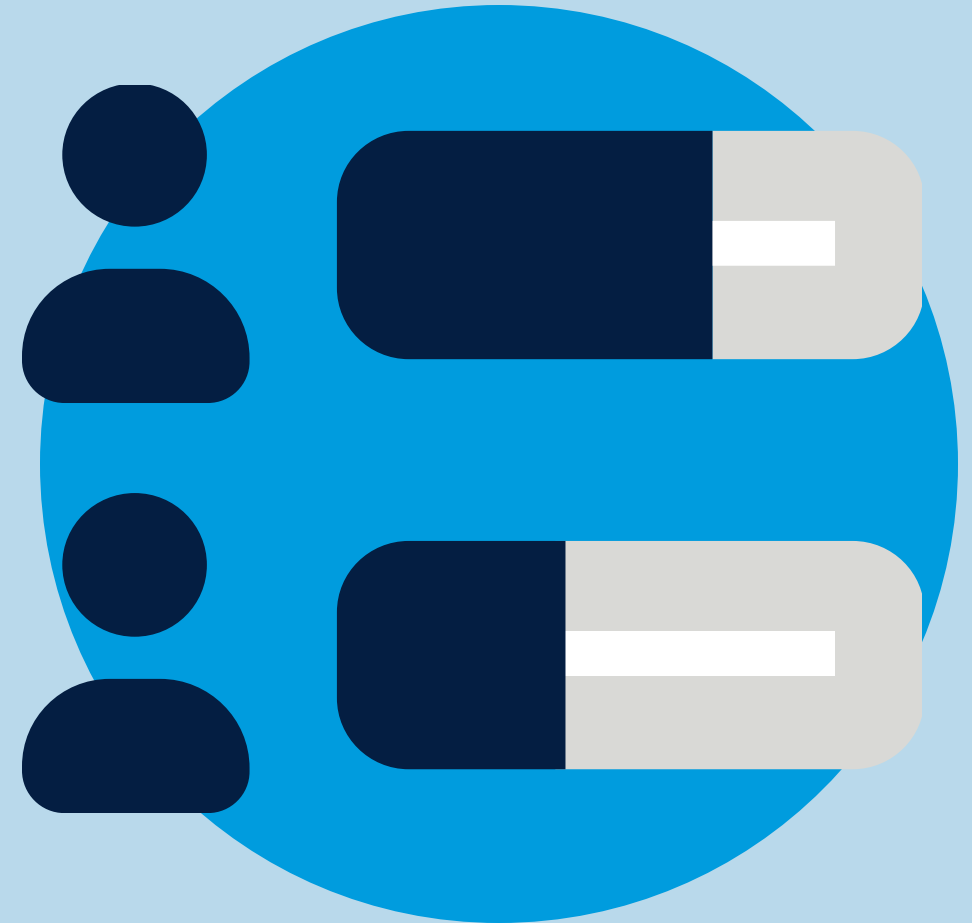
Questions?



Polling Question


#2 Have you done 0D and 2D SAXS?

1. Yes
2. No
3. Both
4. I don't do SAXS.

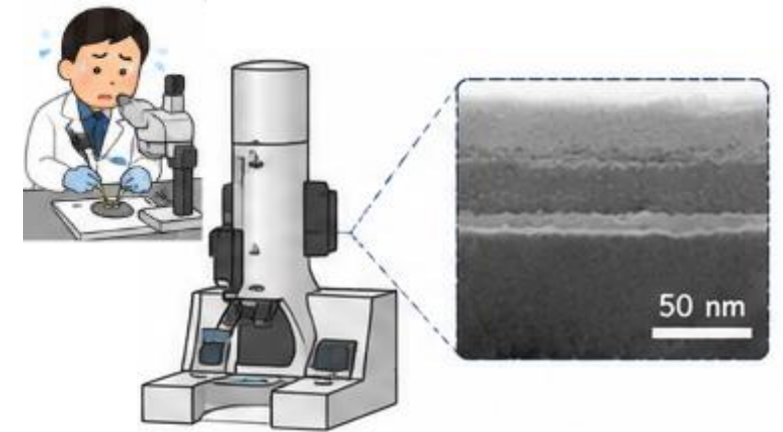
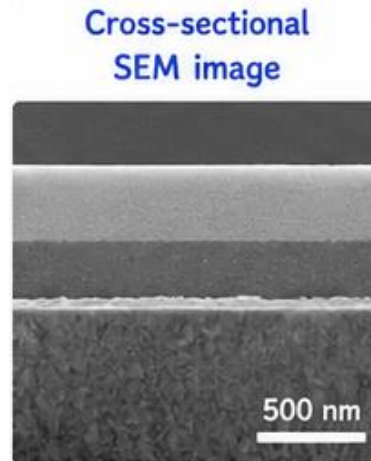
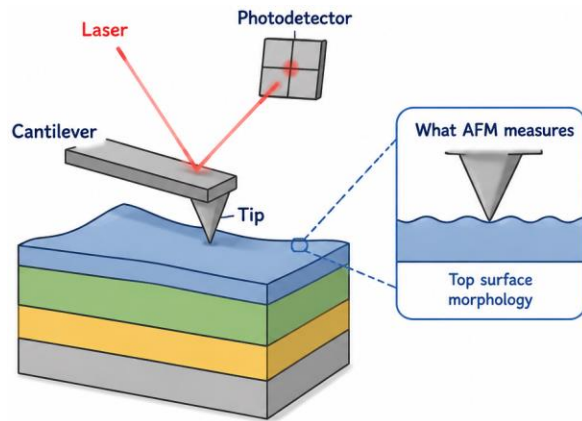


7. XRR fundamentals

Why do we care about surfaces and thin films?

- Semiconductors
 - Magnetic multilayers
 - Optical coatings
 - Organic films and bio membranes
- 
- How thick is the film?
 - Is the interface sharp or diffuse?
 - How smooth is the surface?

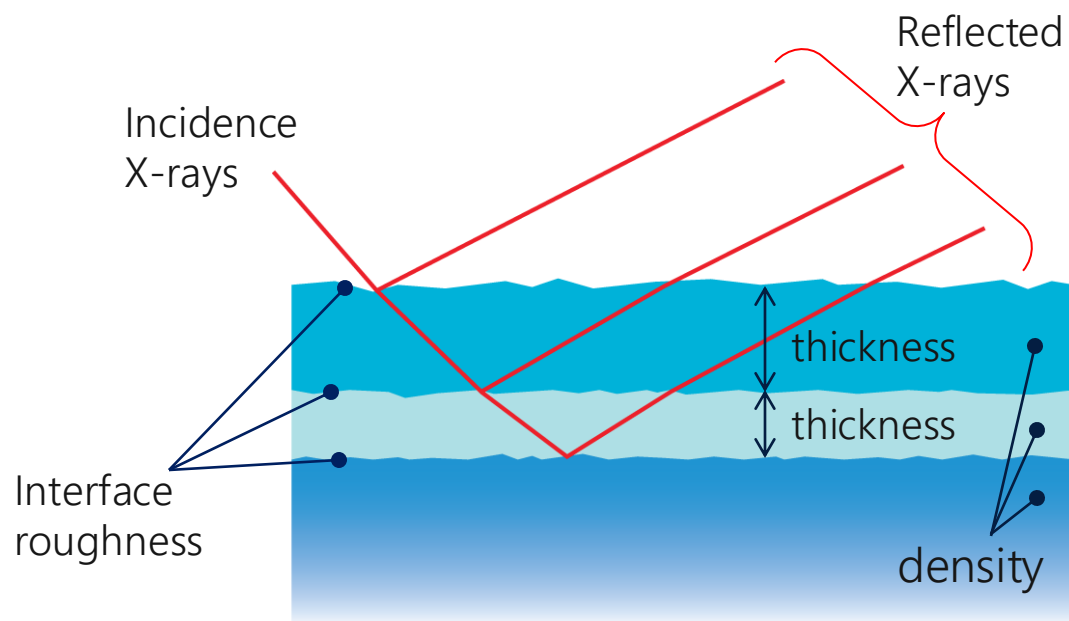
The challenge



- AFM only probes the top surface.
- SEM requires cross-section preparation.
- TEM is destructive and labor-intensive.

XRR non-destructively characterizes the entire thin-film stack over a large area.

What is X-ray Reflectivity?



XRR measures how X-rays reflect from a surface as a function of incident angle.

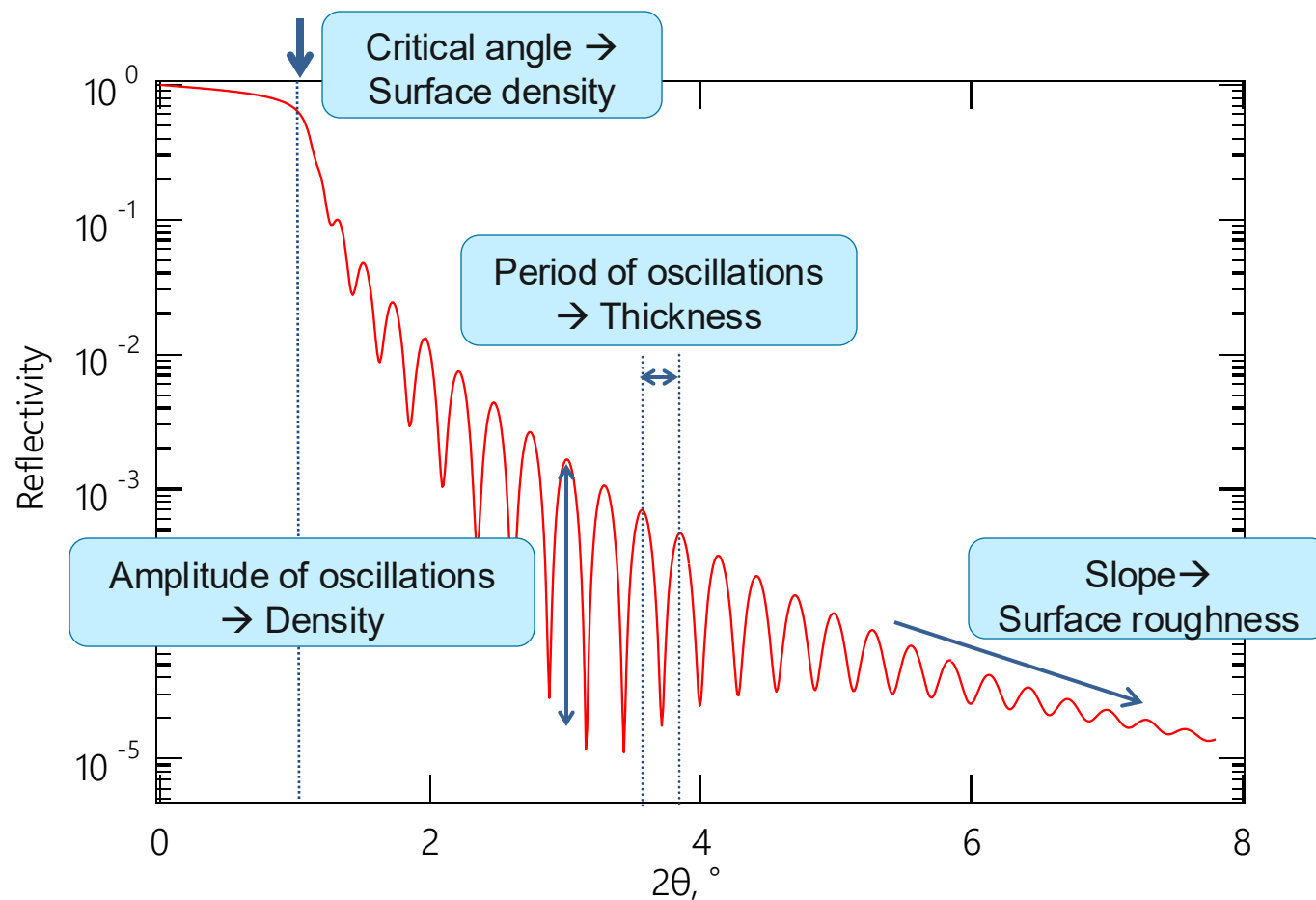
- Thickness (0.1 ~ 1,000nm)
- Density
- Surface & interface roughness (0 ~ 3 nm)
 - $q_{z-max} \sigma_{max} \gg 1$
 - $\sigma_{max} \approx 3 \text{ nm}$

[S. K. Sinha, E. B. Sirota, S. Garoff, H. B. Stanley](#)
["X-ray and neutron scattering from rough surfaces"](#)
[Phys. Rev. B 38, 2297 \(1988\).](#)

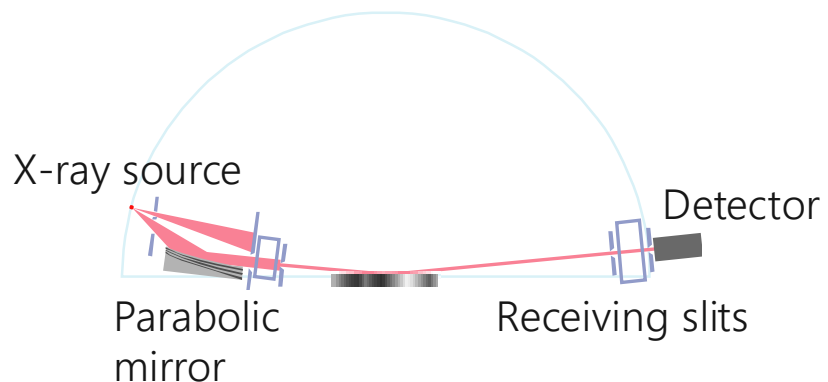
X-ray reflectivity profile

XRR

Specular reflection
 $q \perp$ surface

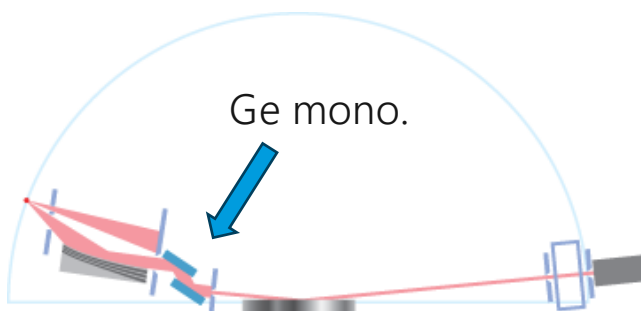


Instrument configurations



- Parallel beam geometry (0.1 ~ 220 nm)

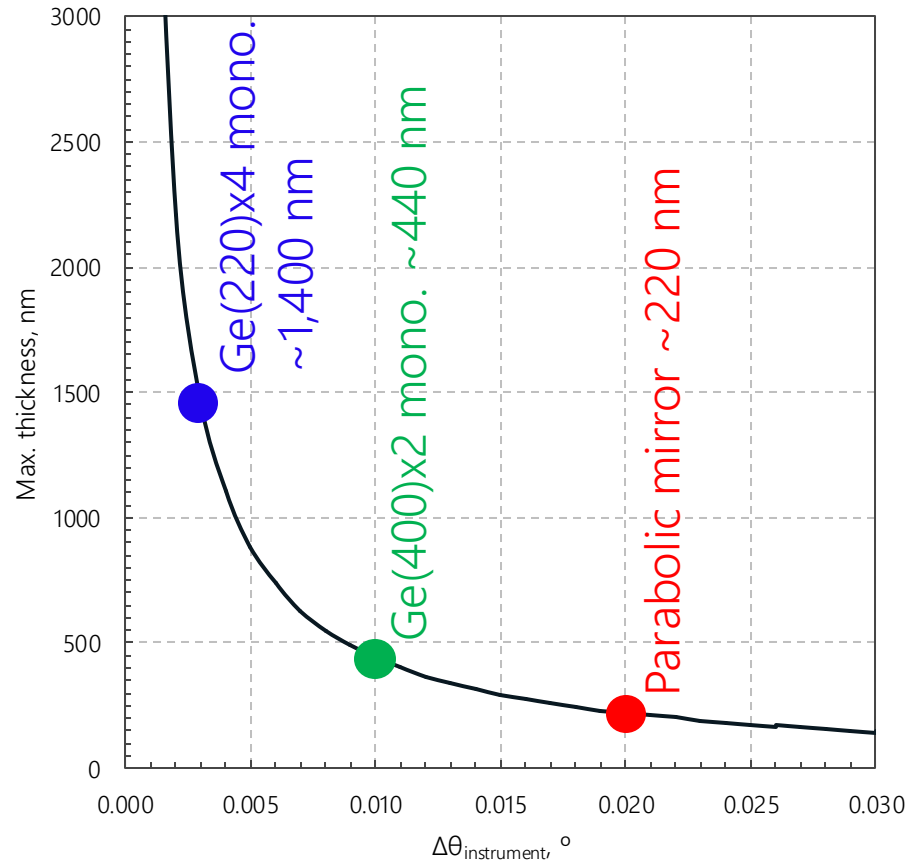
- Parabolic mirror
- Beam divergence: $\Delta\theta \sim 0.02^\circ$



- High resolution geometry (1 ~ 440 nm)

- Parabolic mirror + Ge(400)x2
- Beam divergence: $\Delta\theta \sim 0.01^\circ$

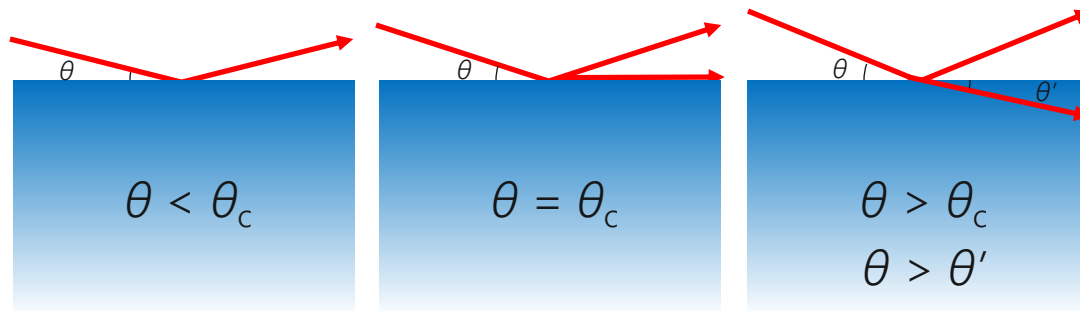
Measurable thickness range



- $\Delta\theta_{\text{thickness}} \approx \frac{\lambda}{2 \cdot t}$
 $\Delta\theta$: fringe spacing
 t : thickness of film (nm)
 λ : wavelength of X-ray 0.15406 (nm)
- Maximum thickness $t_{\text{max.}}$ measured

$$t_{\text{max.}} \approx \frac{\lambda}{2 \cdot \Delta\theta_{\text{instrument}}}$$
 $\Delta\theta_{\text{instrument}}$: instrumental resolution

Critical angle of external total reflection



- Refractive index: $n = 1 - \delta + i\beta$
 - $\delta = \frac{\lambda^2 r_e}{2\pi V_c} \sum_j (Z_j + f_j')$ → Density
 - $\beta = \frac{\lambda^2 r_e}{2\pi V_c} \sum_j (-f_j'')$ → Absorption
 - $\theta_c = \sqrt{2\delta}$ → ~Density

V_c = unit cell volume

Z_j = number of electrons of atom j

f_j' = real anomalous dispersion correction

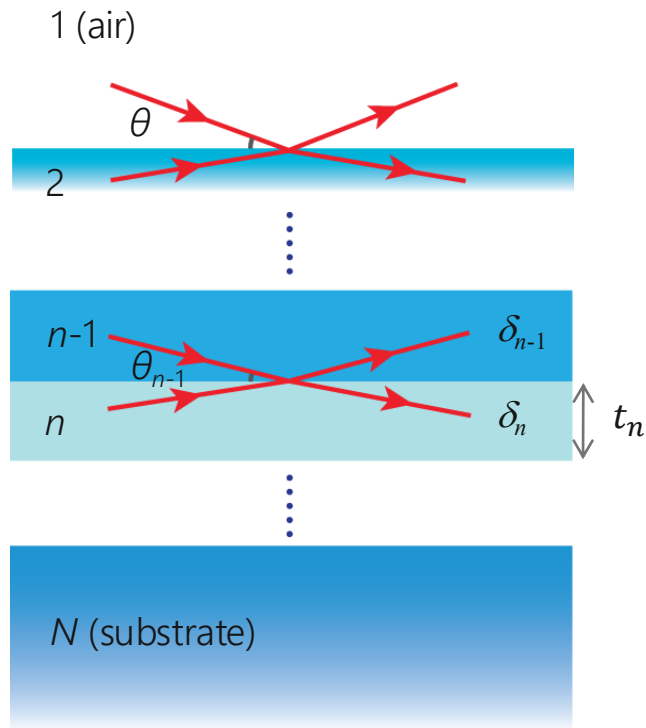
f_j'' = imaginary anomalous dispersion correction

r_e = classical electron radius

θ_c = critical angle for total reflection

Parratt's recursive formula

[L.G.Parratt, Phys.Rev.95\(1954\)359.](#)



- Recursive equation of reflectivity

$$R_{n-1,n} = a_{n-1}^4 \left[\frac{R_{n,n+1} + F_{n-1,n}}{R_{n,n+1} \cdot F_{n-1,n} + 1} \right] \times \exp \left\{ -\frac{1}{2} \sigma_n^2 \left(\frac{4\pi \sin \theta}{\lambda} \right)^2 \right\}$$

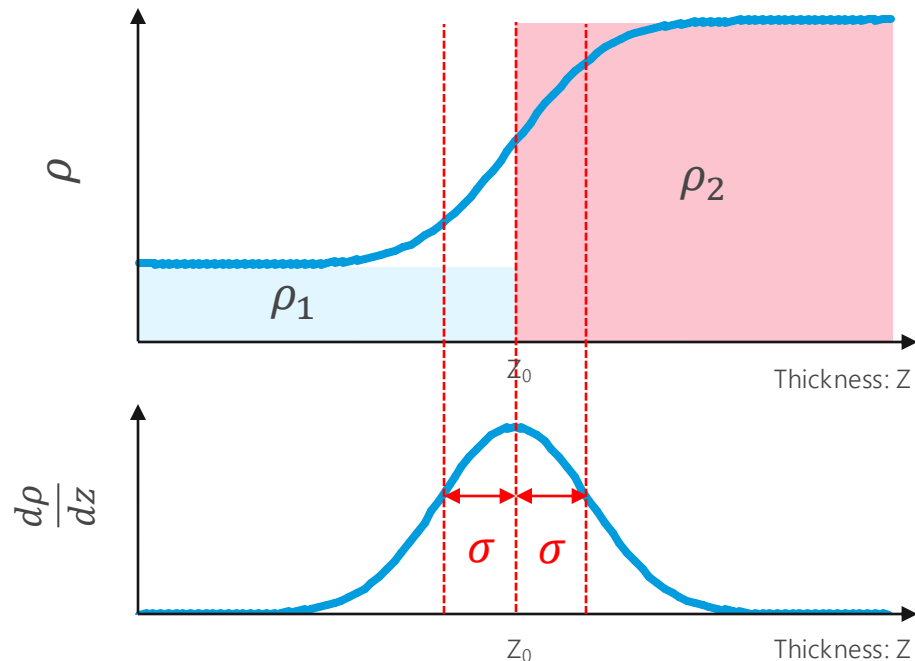
- $a_n = \exp \left(-ik_1 f_n \frac{t_n}{2} \right)$ → Thickness
- $F_{n-1,n} = \frac{f_{n-1} - f_n}{f_{n-1} + f_n}$
- $f_n = \sqrt{\theta^2 - 2\delta_n - 2i\beta_n}$ → Density
- σ_n : roughness of n^{th} interface
- t_n : thickness of n^{th} layer
- θ : Bragg angle
- λ : wavelength of X-ray

Roughness

Density

Nevot-Croce roughness model

[L. Nevot & P. Croce, Rev.Phys.Appl.15\(1980\)761.](#)

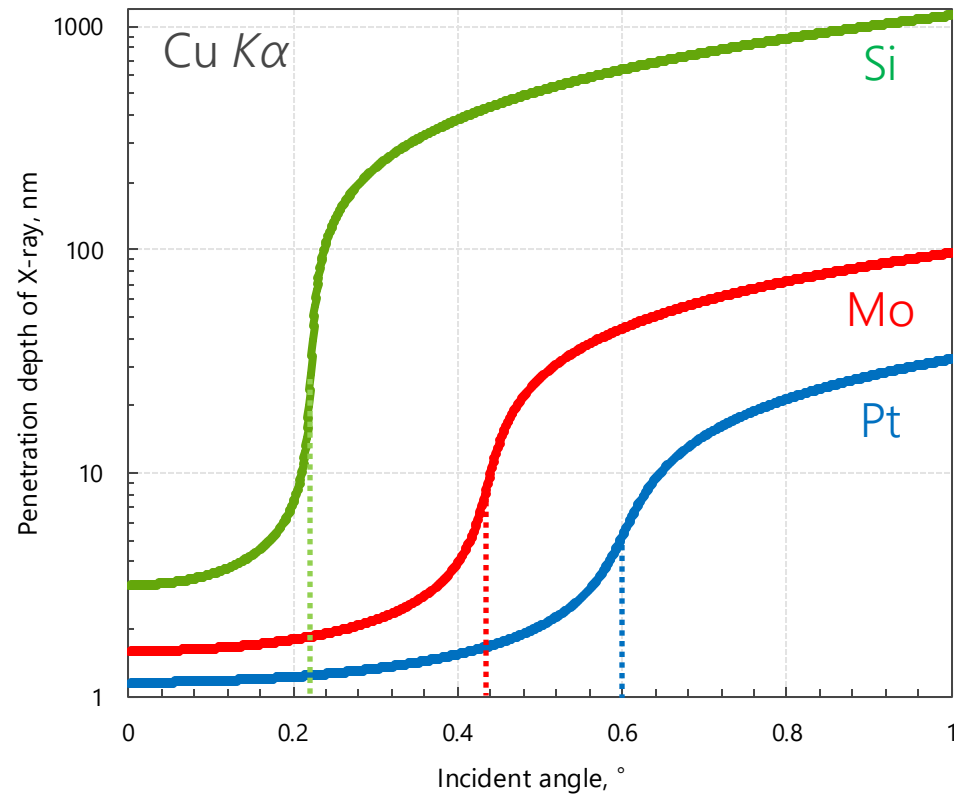


- Electron density profile across an interface is described by an error function.
 - $\rho(z) = \rho_{\text{avg}} + \frac{\Delta\rho}{2} \text{erf}\left(\frac{z-z_0}{\sqrt{2}\sigma}\right)$ → Roughness
- Roughness parameter σ corresponds to the standard deviation of the Gaussian that is the 1st derivative of the density profile.

- $\frac{d\rho}{dz} = \frac{\Delta\rho}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(z-z_0)^2}{2\sigma^2}\right]$

Roughness

Penetration depth



- Penetration depth D around θ_c

- $D = \frac{\lambda}{4\pi B}$

- $2B^2 = (-\theta^2 + 2\delta) + [(\theta^2 - 2\delta)^2 + 4\beta^2]^{1/2}$

Density

Absorption

- θ_c

- Si ($\rho = 2.33 \text{ g/cm}^3$): 0.22°

- Mo ($\rho = 10.28 \text{ g/cm}^3$): 0.44°

- Pt ($\rho = 21.46 \text{ g/cm}^3$): 0.60°

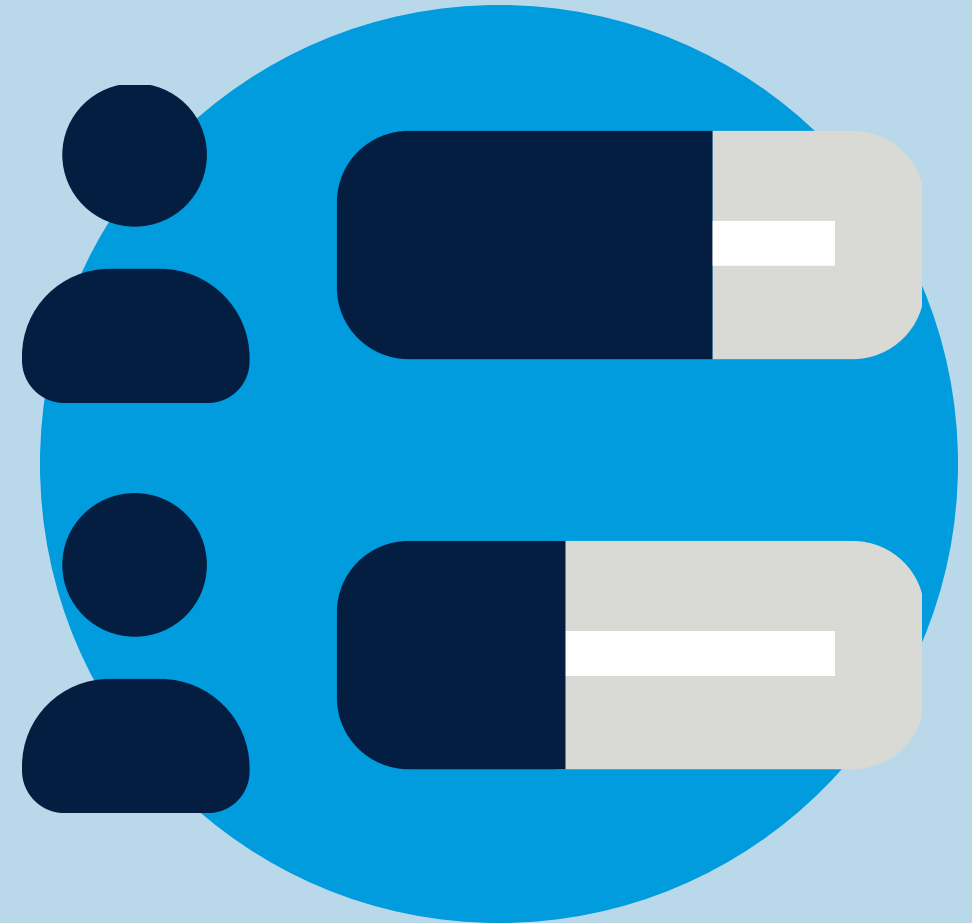
Questions?



Polling Question

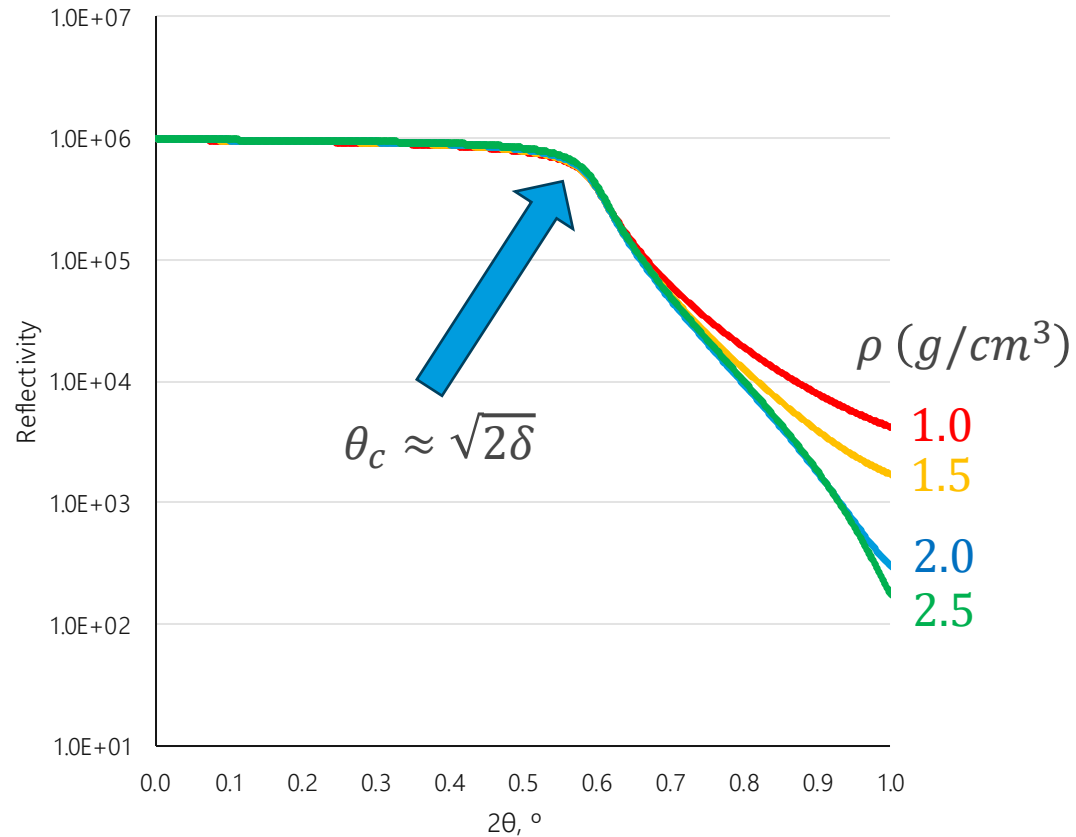
#3 Had you previously considered XRR modeling to be nanostructural analysis?

1. Yes
2. No



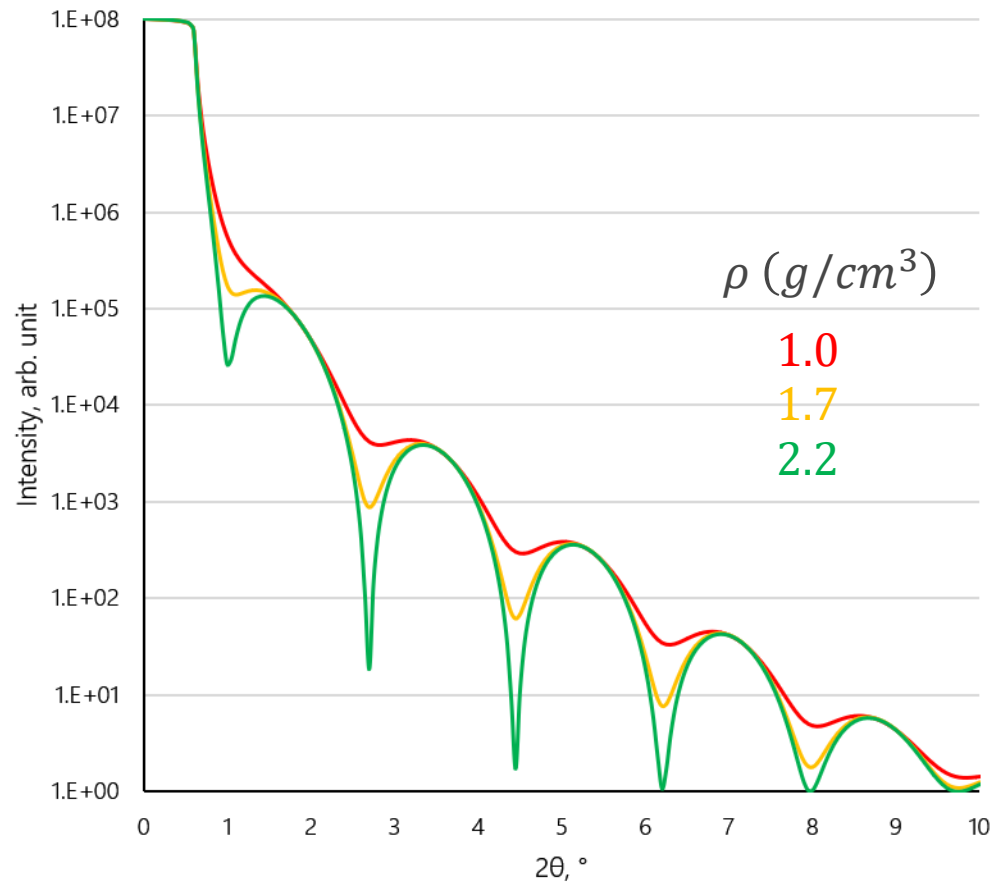
8. *XRR modeling & fitting*

Critical angle for total reflection



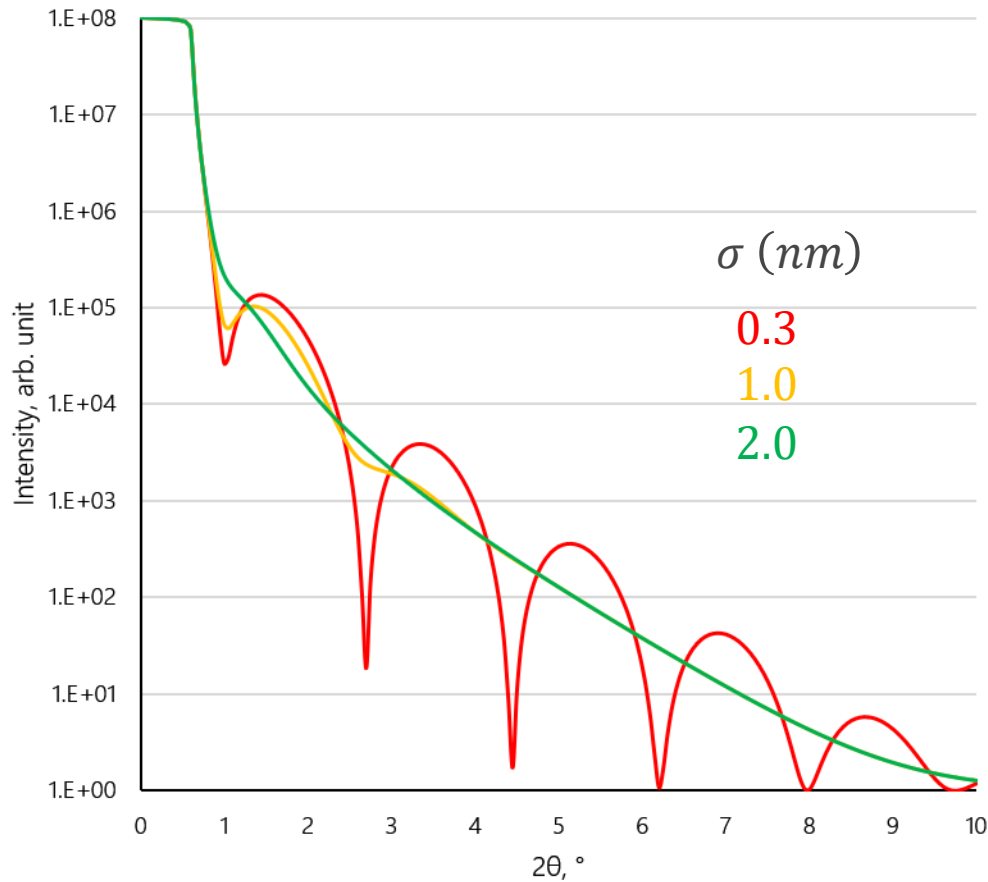
- SiO₂(5nm)/GaAs sub.
- SiO₂ with different densities show the same θ_c .
- 5 nm-thick SiO₂ is too thin to show θ_c .

Profile vs. density of SiO₂ layer



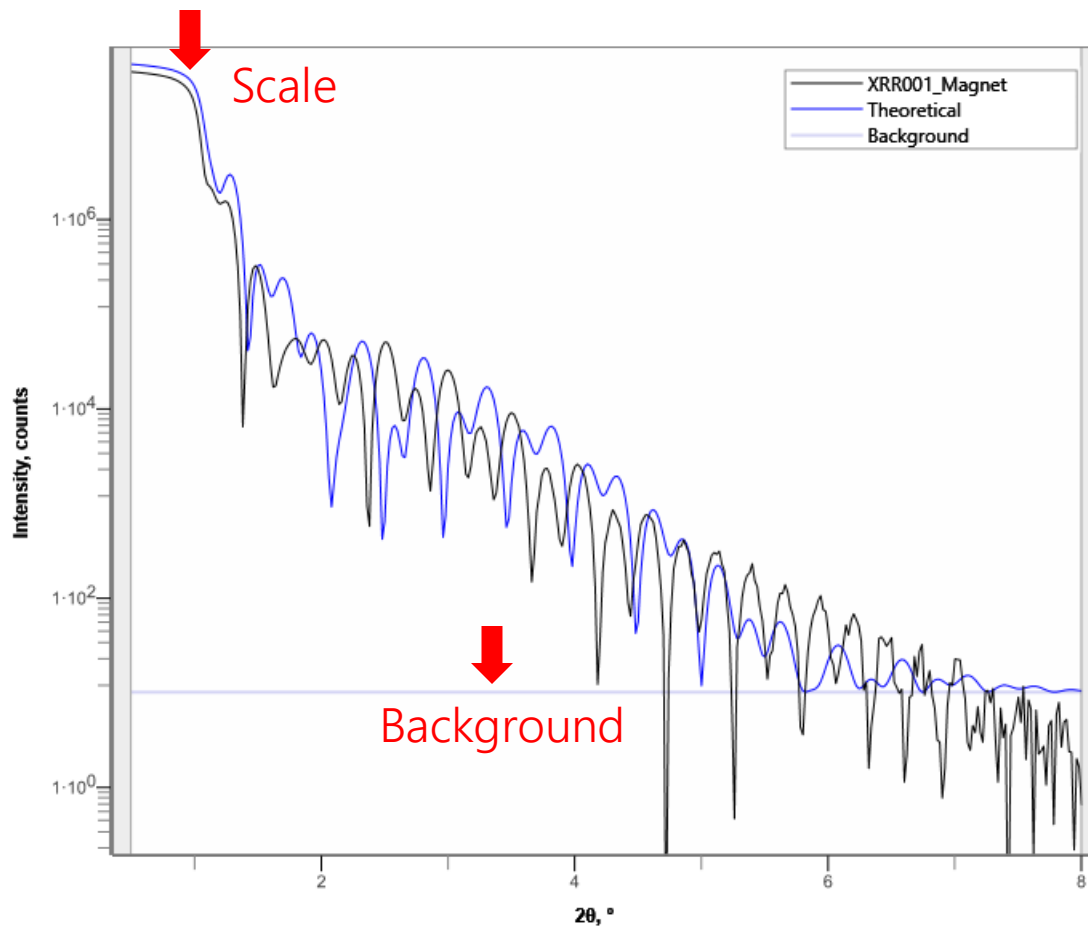
- SiO₂(5nm)/GaAs sub.
- Roughness: 0.3 nm.
- Density of SiO₂ changes amplitude of the thickness fringes.

Profile vs. roughness of SiO₂ layer



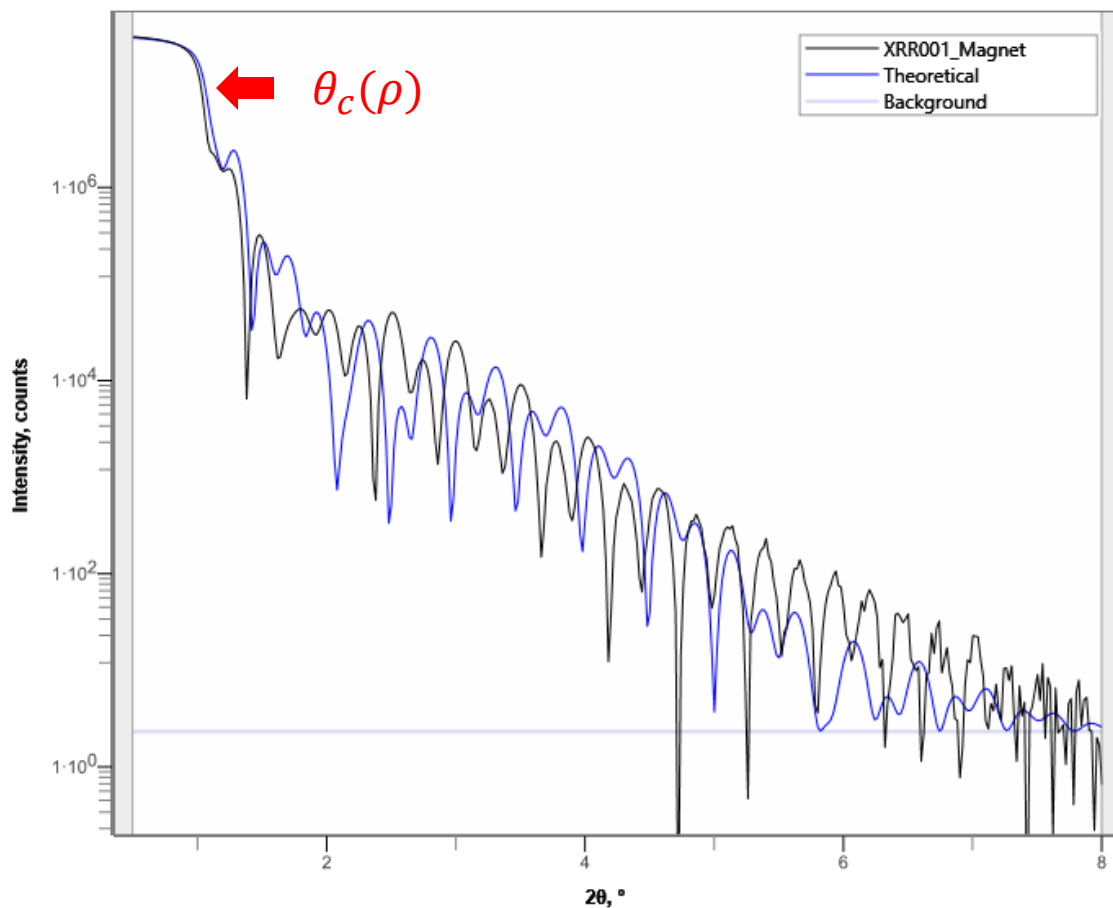
- SiO₂(5nm)/GaAs sub.
- Density: 2.2 g/cm³.
- σ changes maximum 2θ angle for fringes.

XRR profile fitting



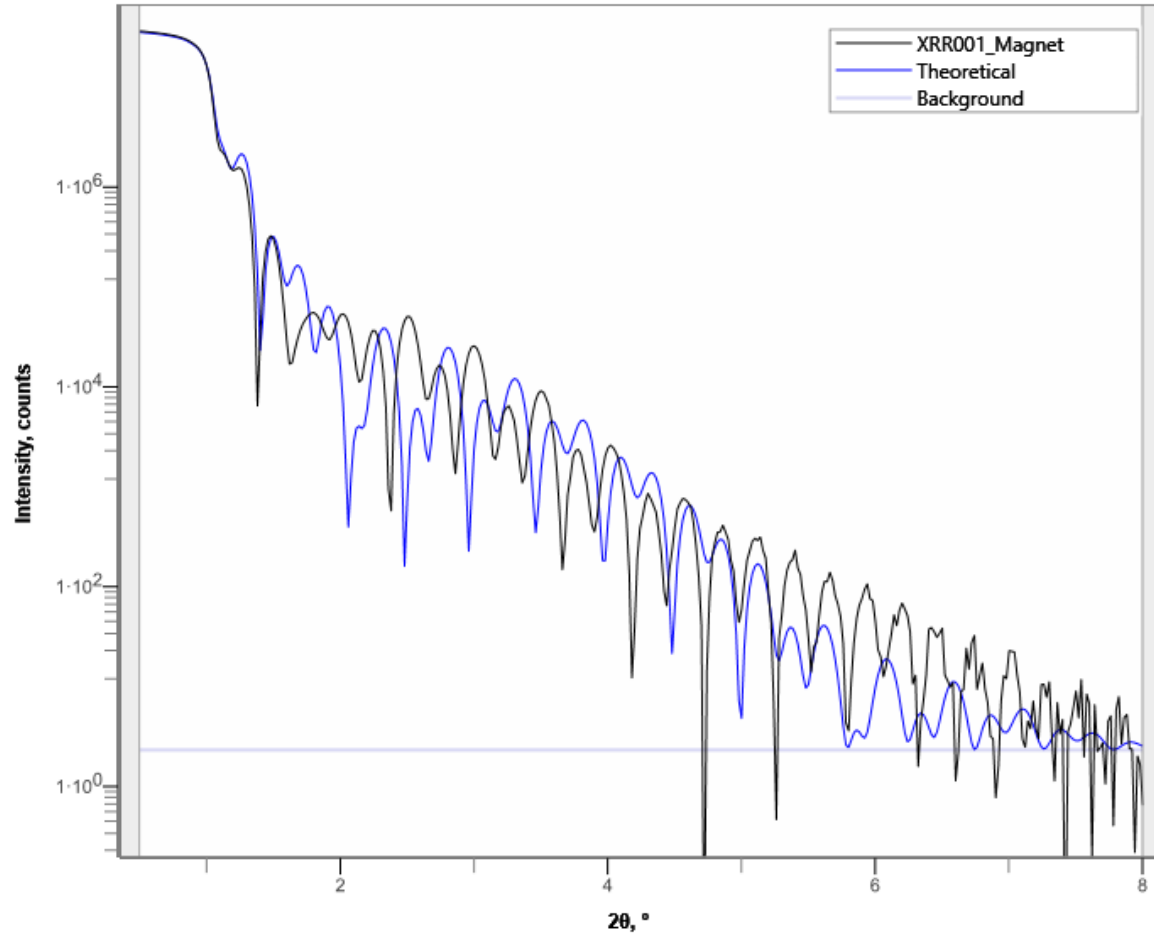
Layer	Thickness [nm]	Density [g/cm ³]	Roughness [nm]
TaO	2.30	8.70	0.50
Ta	16.00	16.70	0.50
NiFe	16.00	8.70	0.50
Si sub.	n.a.	2.33	0.50

XRR profile fitting: step (1)

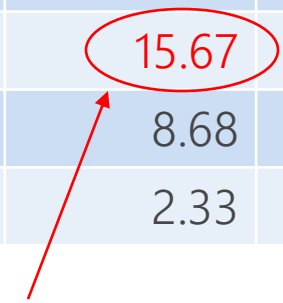


Layer	Thickness [nm]	Density [g/cm ³]	Roughness [nm]
TaO	2.30	8.70	0.50
Ta	16.00	16.70	0.50
NiFe	16.00	8.70	0.50
Si sub.	n.a.	2.33	0.50

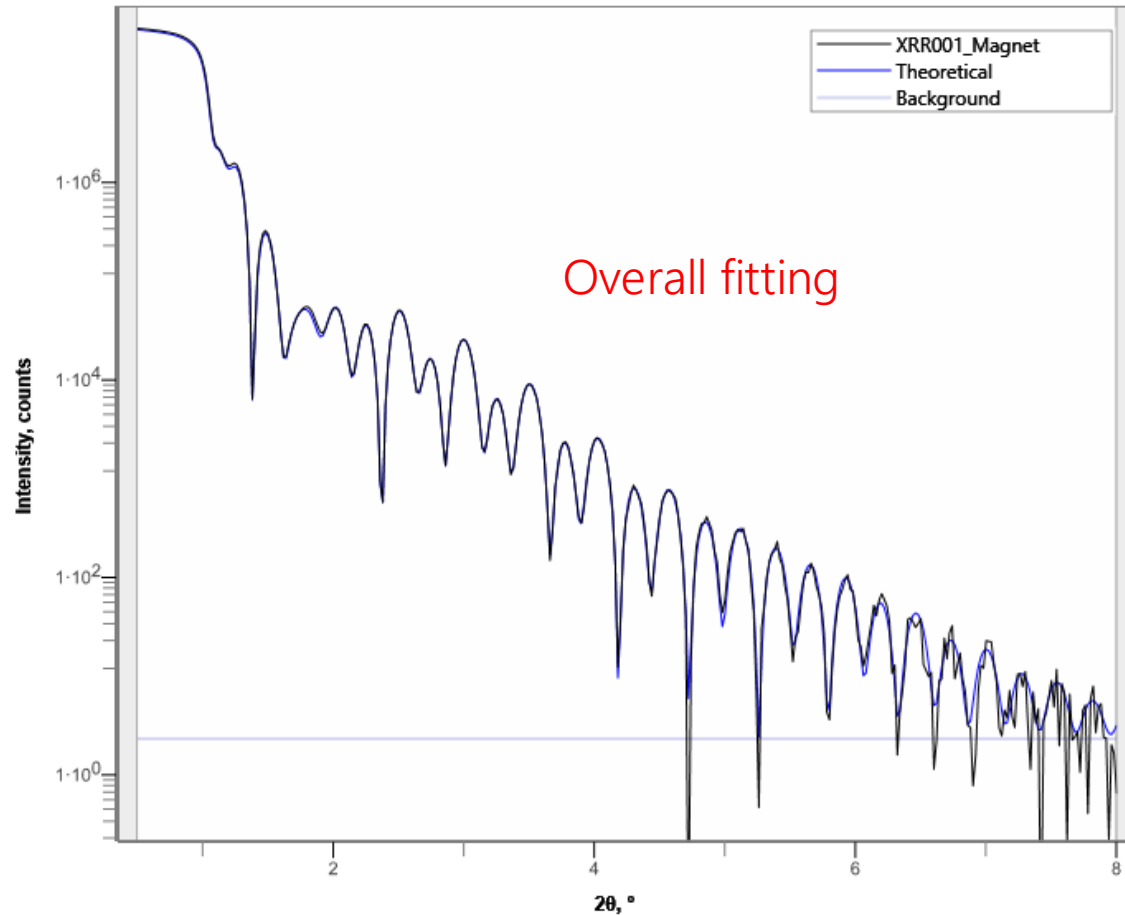
XRR profile fitting: step (2)



Layer	Thickness [nm]	Density [g/cm ³]	Roughness [nm]
TaO	2.30	8.70	0.50
Ta	16.00	15.67	0.50
NiFe	16.00	8.68	0.50
Si sub.	n.a.	2.33	0.50



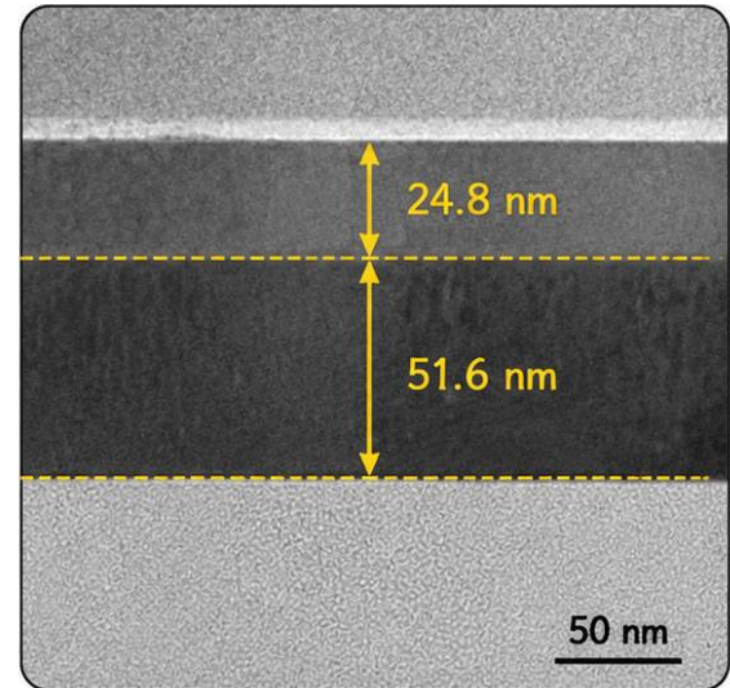
XRR profile fitting: step (3)



Layer	Thickness [nm]	Density [g/cm ³]	Roughness [nm]
TaO	2.80	8.11	0.72
Ta	17.10	15.67	0.38
NiFe	15.30	8.61	0.56
Si sub.	n.a.	2.33	0.42

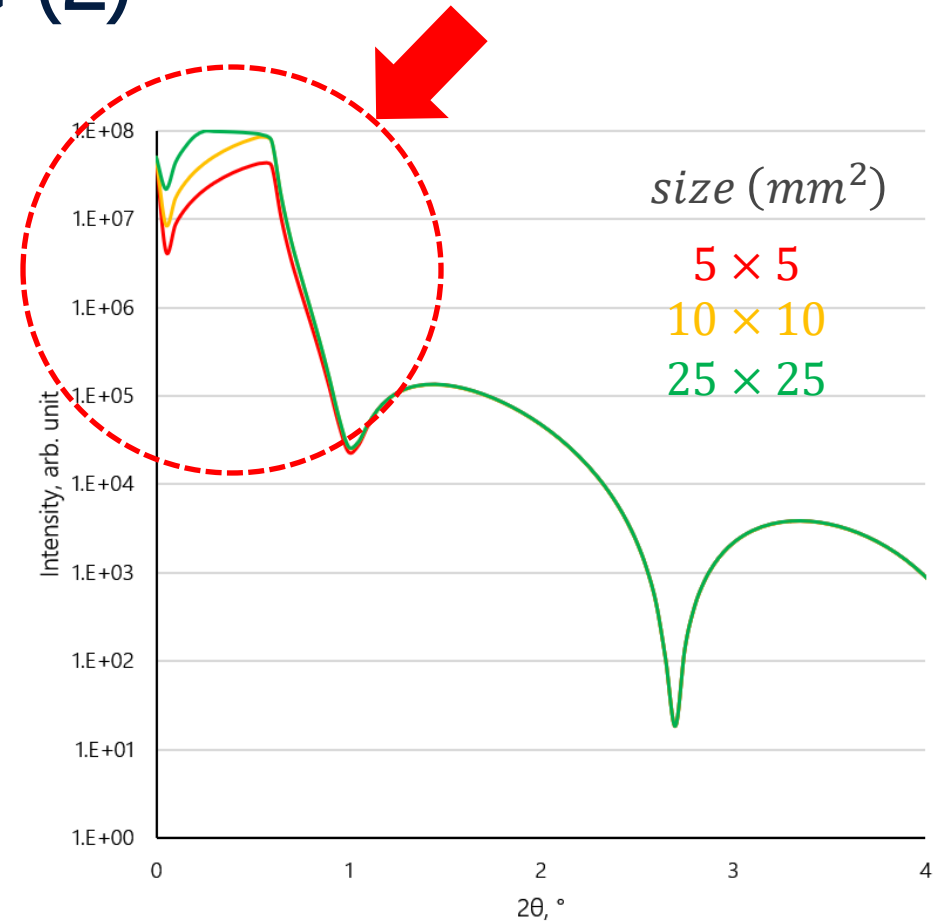
When the fit doesn't converge (1)

- Try TEM to get an idea about thickness(es).



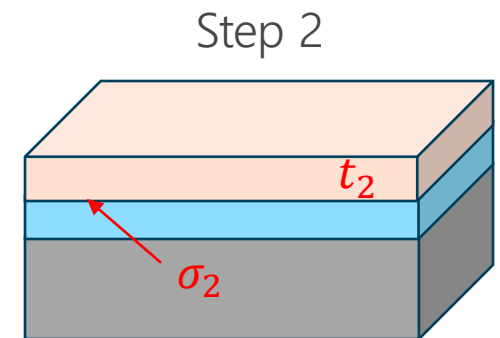
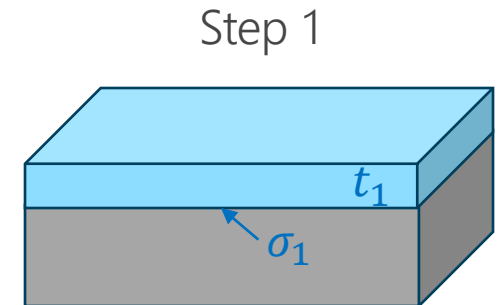
When the fit doesn't converge (2)

- Prepare a sample with surface area larger than 10x10 mm².



When the fit doesn't converge (3)

- Characterize individual layers before building the full stack



Summary

- XRR is a non-destructive technique for characterizing thin films and interfaces, providing thickness, density, and surface/interface roughness with nanometer-scale sensitivity.
- Instrument configuration determines the measurable thickness range, extending from a few nanometers to over one micrometer with high-resolution optics.

Questions?





We'll follow up with your questions.



Recording will be available tomorrow.



More new content is coming soon.

Capillary

- Fused silica (quartz)
 - Low background
 - Excellent uniformity
 - Low parasitic scattering
- Borosilicate
 - B, Na, Al, K
 - Diffuse scattering
 - Higher background
 - Parasitic scattering

Distance to nearest neighbor: d_{nn}

- Average center-to-center distance between neighboring particles
 - $\therefore d_{nn} \approx \left(\frac{\pi\sigma^3}{6\eta}\right)^{\frac{1}{3}}$
 - d_{nn} is a function of σ and η .

Example of δ , β and θ_c for Cu-K α

	ρ [g/cm ³]	δ [10 ⁻⁶]	β [10 ⁻⁷]	θ_c [°]
Si	2.33	7.57	1.75	0.223
GaAs	5.32	14.5	4.20	0.309
InP	4.79	13.5	1.15	0.298
Ge	5.33	14.5	4.17	0.309
C	3.52	11.3	16.9	0.272
SiO ₂	2.65	8.57	1.11	0.237
HfO ₂	10.11	25.05	15.51	0.405
Ta ₂ O ₅	9.07	22.8	14.1	0.387
TiN	5.39	16.2	10.2	0.326
TaN	14.3	34.6	24.9	0.477
Cu	8.94	24.3	5.31	0.400
W	19.26	45.6	37.4	0.547
Al	2.70	8.47	1.58	0.236

